# Formalizing Date Arithmetic and Statically Detecting Ambiguities for the Law

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Cambium seminar
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# Some Catala News, and an Introduction

# Legal implementations

Some legal implementations are critical software: taxes, benefits

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- a DSL for computational laws

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### Catala

- a DSL for computational laws
- providing transparency
- easing maintenance
- through interdisciplinary work

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## Industrialization ongoing at Inria

- Recognized as a priority program of support to public policies
- From 1 to 3 research engineers in the coming months (+ Denis)

# Handling Date Computations in Catala

# Computing dates

```
$ date -d "2024-01-31 + 1 month" +%F
```

# Computing dates

```
$ date -d "2024-01-31 + 1 month" +%F
2024-03-02
```

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```

Non-monotonic behavior?!

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$\Longrightarrow$ Formal, flexible semantics required!

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$\Longrightarrow$ Formal, flexible semantics required! Focus on Gregorian calendar.

# Outline

# Semantics

# Semantics – Values

| | | | |
|---|---|---|---|
| values | $v$ | $::=$ | $(y, m, d) \mid \perp$ |
| date unit | $\delta$ | $::=$ | $y \mid m \mid d$ |
| expressions | $e$ | $::=$ | $v \mid e \ +_{\delta} \ n$ |

# Semantics – Values

$$\begin{array}{llll} \text{values} & v & ::= & (y, m, d) \mid \bot \\ \text{date unit} & \delta & ::= & y \mid m \mid d \\ \text{expressions} & e & ::= & v \mid e \ +_\delta \ n \end{array}$$

$$\text{nb\_days}(y, m) = \begin{cases} 29 \text{ if } m = 2 \land \text{is\_leap}(y) \\ 28 \text{ if } m = 2 \land \neg\text{is\_leap}(y) \\ 30 \text{ if } m \in \{\, \texttt{Apr}, \texttt{Jun}, \texttt{Sep}, \texttt{Nov} \,\} \\ 31 \text{ otherwise} \end{cases}$$

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$$\frac{day > \text{nb\_days}(y, m)}{(y, m, day) +_d n \rightarrow \bot}\ \textsc{Add-Days-Err2}$$

# Semantics – some cases of month addition

$$\textsc{Add-Month}\quad \frac{1 \leq m + n \leq 12}{(y, m, d) +_m n \to (y, m + n, d)}$$

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$$\frac{1 \leq m + n \leq 12}{(y, m, d) +_m n \to (y, m + n, d)}\ \textsc{Add-Month}$$

$$\frac{m + n > 12}{(y, m, d) +_m n \to (y + 1, m, d) +_m (n - 12)}\ \textsc{Add-Month-Over}$$

## Semantics – some cases of month addition

$$\frac{1 \leq m + n \leq 12}{(y, m, d) +_m n \rightarrow (y, m + n, d)}\ \textsc{Add-Month} \qquad \frac{m + n > 12}{(y, m, d) +_m n \rightarrow (y + 1, m, d) +_m (n - 12)}\ \textsc{Add-Month-Over}$$

Similar cases for ADD-MONTH-UNDER, year, day addition.

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$$(2024, 01, 31) +_m 1 \to (2024, 02, 31)$$

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$$\begin{aligned} \mathrm{rnd}_\uparrow(2024, 02, 31) &= (2024, 03, 01) \\ \mathrm{rnd}_\downarrow(2024, 02, 31) &= (2024, 02, 29) \\ \mathrm{rnd}_\perp(2024, 02, 31) &= \perp \end{aligned}$$

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Coreutils-like rounding not defined here

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Round-Up

$$\frac{d > \mathrm{nb\_days}(y, m) \qquad (y, m, d) +_m 1 \xrightarrow{*} (y', m', d')}{\mathrm{rnd}_\uparrow(y, m, d) \rightarrow (y', m', 1)}$$

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$$\frac{\textsc{Round-Up}\quad d > \text{nb\_days}(y, m) \qquad (y, m, d) +_m 1 \xrightarrow{*} (y', m', d')}{\text{rnd}_\uparrow(y, m, d) \rightarrow (y', m', 1)}$$

$$\frac{\textsc{Round-Err2}\quad d > \text{nb\_days}(y, m)}{\text{rnd}_\perp(y, m, d) \rightarrow \perp}$$

**Date-period addition**

Given a period $(ys, ms, ds)$:

$$e +_r (ys, ms, ds) ::= \mathrm{rnd}_r((e +_y ys) +_m ms) +_d ds$$

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Avoids double rounding

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## Ambiguous expression

A date expression $e$ is ambiguous iff $\mathrm{rnd}_\perp(e) \xrightarrow{*} \perp$

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iff roundings $e$ yield different values

# Formalized Properties

## Non-properties

### Commutativity of addition

$$(2024, 03, 31) +_{\uparrow} 1\text{m} +_{\uparrow} 1\text{d} = (2024, 05, 01) +_{\uparrow} 1\text{d} = (2024, 05, 02)$$

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**“Associativity” of addition**

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$$(2024, 03, 31) +_{r} 2\mathrm{m} = (2024, 05, 31)$$

# Formalized properties

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**Well-formedness**

For any date $d$, any period $p$, any value $v$, and $r \in \{\downarrow, \uparrow\}$, we have:

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For any dates $d_1, d_2$, period $p$, $r \in \{\downarrow, \uparrow\}$, if $d_1 < d_2$, then $d_1 +_r p \leq d_2 +_r p$

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**Loose bound in conclusion of monotonicity**

$(2024, 03, 30) +_\downarrow 1m = (2024, 04, 30) = (2024, 03, 31) +_\downarrow 1m$

# Formalized properties (II)

**Rounding is monotonic**

For all date *d*, period *p*:

1. $d +_{\downarrow} p \leq d +_{\uparrow} p$
2. $d +_{\perp} p \neq \perp \Rightarrow d +_{\downarrow} p = d +_{\uparrow} p = d +_{\perp} p$

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**Equivalence of year and month addition**

For all date $d$, for all integer $n$, $d +_y n = d +_m (12 * n)$.

# Formalized properties (III)

## Ambiguous month addition

For all valid date $d$, integer $n$ such that $d +_m n \xrightarrow{*} (y, m, day)$:

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$\Longrightarrow$ core result needed for our static analysis

# Rounding-insensitivity Static Analysis

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To reduce the need for costly legal interpretations

# Rounding-insensitivity Static Analysis

Abstracting dates in a fixed rounding mode

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$\mathbf{d}(d_1) \in [1, 31] \wedge \mathbf{m}(d_1) \in [1, 12] \wedge \mathbf{y}(d_1) = 2024$: all valid dates of 2024

## YMD domain – month addition

**Goal**

Given a rounding mode, compute resulting dates from $d^{\#} +_m^{\#} n$, where $d^{\#}$ represents a set of dates.

Soundly derived from the ambiguous addition theorem.

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Algorithm: compute resulting month, year, then 4 cases:

- No rounding,
- Rounding, 30-day month,
- Rounding, non-leap years 28 Feb,
- Rounding, leap years, 29 Feb.

Partitioning used in practice.

# YMD domain – month addition (II)

```
type case = expr * state
type cases = case list
let switch abs =
  List.map (fun (cond : expr, k : state -> case) -> k (assume cond abs))

let add_months (r: rnd) ((d, m, y): var^3) (nb_m: int) (abs: state): cases =
  let res_m: expr = 1 + (m - 1 + nb_m) % 12 in
  let res_y: expr = y + (m - 1 + nb_m) / 12 in
  switch abs
  [
    d > 30 && is_one_of res_m [Apr;Jun;Sep;Nov],
      round r 30 res_m res_y;
    d > 28 && res_m = Feb && not (is_leap res_y),
      round r 28 res_m res_y;
    d > 29 && res_m = Feb && is_leap res_y,
      round r 29 res_m res_y;
    mk_true,
      mk_date d res_m res_y
  ]
```

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`date d1 = rand_date(); date d2 = d1 + 1 month;` rounding down.

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$$\mathrm{d}(d1) = 31,\ \mathrm{m}(d1) \in \{\mathtt{Mar}, \mathtt{May}, \mathtt{Aug}, \mathtt{Oct}\},\ \mathrm{m}(d2) = \mathrm{m}(d1) + 1, \mathrm{y}(d2) = \mathrm{y}(d1)$$

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$$\mathrm{d}(d1) = 31,\ \underbrace{\mathrm{m}(d1) \in \{\,\mathtt{Mar}, \mathtt{May}, \mathtt{Aug}, \mathtt{Oct}\,\}}_{\text{Bounded set of ints}},\ \mathrm{m}(d2) = \mathrm{m}(d1) + 1, \mathrm{y}(d2) = \mathrm{y}(d1)$$

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[1] Actually, $12\,\mathrm{y}(\mathtt{d1}) + \mathrm{m}(\mathtt{d1}) \leq 12\,\mathrm{y}(\mathtt{d2}) + 11 \wedge 12\,\mathrm{y}(\mathtt{d2}) \leq 12\,\mathrm{y}(\mathtt{d1}) + \mathrm{m}(\mathtt{d1}) + 1$

# Rounding-insensitivity Static Analysis

Lifting to both rounding modes

# Back to rounding-insensitivity detection

- Semantics on product programs with both rounding modes.

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- Inspired by Delmas, Ouadjaout, and Miné. “Static Analysis of Endian Portability by Abstract Interpretation”. SAS 2021.

# Abstract double semantics

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- 30-day month

$$\mathrm{d}(d1) = 31, \mathrm{m}(d1) \in \{\mathtt{Mar}, \mathtt{May}, \mathtt{Aug}, \mathtt{Sep}\}$$

$$\downarrow\mathrm{d}(d2) = 30, \downarrow\mathrm{m}(d2) \in \{\mathtt{Apr}, \mathtt{Jun}, \mathtt{Sep}, \mathtt{Nov}\}, \downarrow\mathrm{m}(d2) = \mathrm{m}(d1) + 1$$

$$\uparrow\mathrm{d}(d2) = 1, \uparrow\mathrm{m}(d2) \in \{\mathtt{May}, \mathtt{Jul}, \mathtt{Oct}, \mathtt{Dec}\}, \uparrow\mathrm{m}(d2) = \mathrm{m}(d1) + 2$$

$$\downarrow\mathrm{y}(d2) = \uparrow\mathrm{y}(d2) = \mathrm{y}(d1)$$

# Implementation into Mopsa

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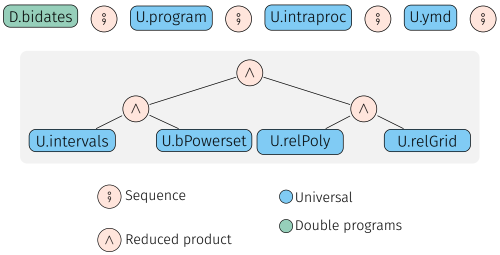

# Extracted sample from French housing benefits

```
date current = rand_date();
date birthday = rand_date();
date intermediate = birthday + [2 years, 0 months, 0 days];
date limit = first_day_of(intermediate);
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```
5: assert(sync(current < limit));
               ^^^^^^^^^^^^^^^

Desynchronization detected: (current < limit). Hints:
↑month(limit) = 3, ↑day(limit) = 1, ↓month(limit) = 2, ↓day(limit) = 1,
↑month(intermediate) = 3, ↑day(intermediate) = 1, ↓month(intermediate) = 2,
↓day(intermediate) = 28, month(birthday) = 2, day(birthday) = 29,
year(birthday) =[4] 0, month(current) = 2, day(current) = [1,29],
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**Computed, actual counter-example**

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# Case Study: French Housing Benefits

**Contributions to Catala**

- Date-rounding library `dates-calc`

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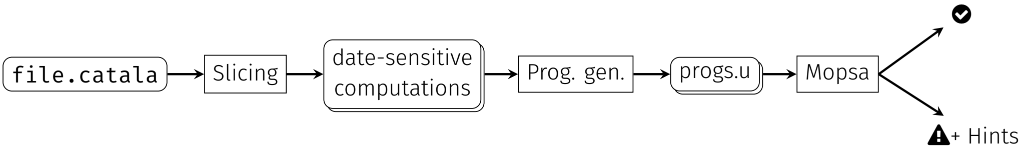
## French Housing Benefits

20,000 Loc of Catala code (including text spec.)

# Date ambiguity detection pipeline



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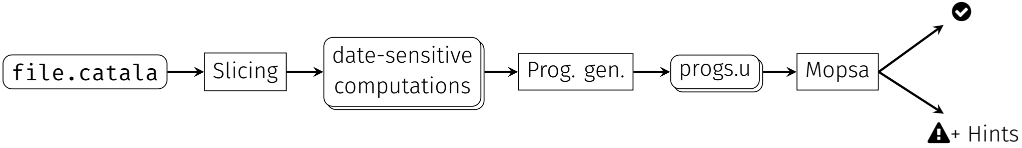


2 rounding-sensitive cases detected

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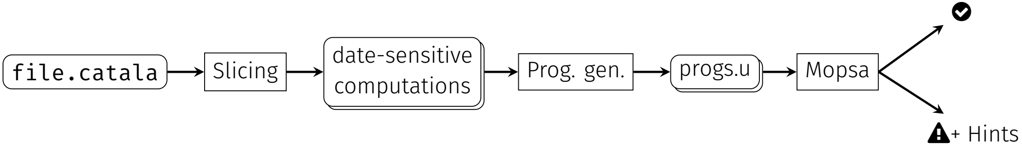


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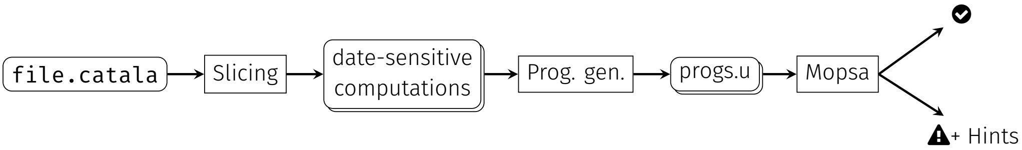


**2 rounding-sensitive cases detected**

**Intra-scope extraction for now**

**Manual inter-scope extraction**

16 additional cases:

- 10 can be proved safe (assuming `current_date` $\geq$ `2023`)
- Other are real issues

# Conclusion

# Related Work

**Survey of implementations**

- Java, `boost` round down
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# Related Work

## Survey of implementations

- Java, **`boost`** round down
- Python **`stdlib`**: no month addition
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## Floating-point arithmetic

- FP widely used & more complex!
- Different rounding modes
- No analysis of rounding-sensitivity?

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“**A**utomatic **V**erification **o**f **Cat**ala programs” (AVoCAT) project