How to prove that you need Cake?

Based on PureCake A Verified Compiler for a Lazy Functional Language

17th Novembre 2023 — Cambium Seminar

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Gordon Sau *University of New South Wales*

Implementing MyCriticalSoftware

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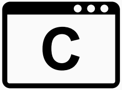

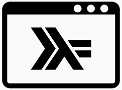

Implementing MyCriticalSoftware

?

Language guarantees

Implementing MyCriticalSoftware

C

?

ML

type safety
memory safety

Implementing MyCriticalSoftware

C	ML	Haskell
?	type safety	type safety
	memory safety	memory safety
		purity vs. I/O
		ref. transparency
		laziness
		free theorems

Compiling MyCriticalSoftware

Compiler guarantees

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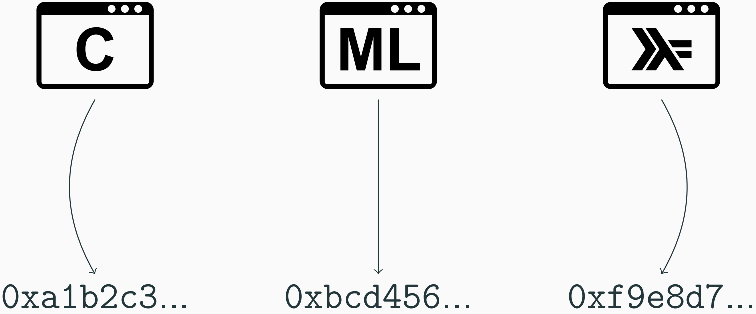

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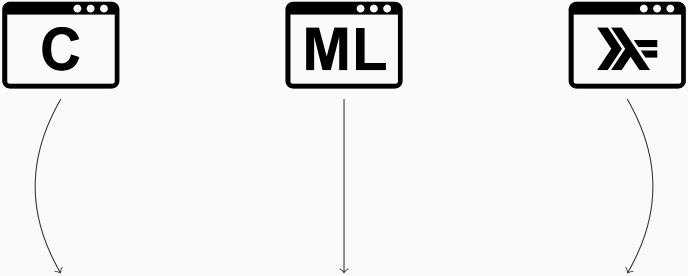

`0xa1b2c3...`   `0xbcd456...`   `0xf9e8d7...`

?   ?   ?

Compiler guarantees

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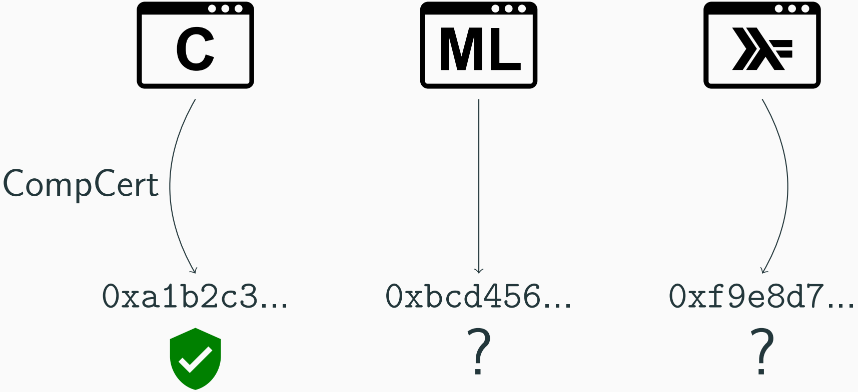

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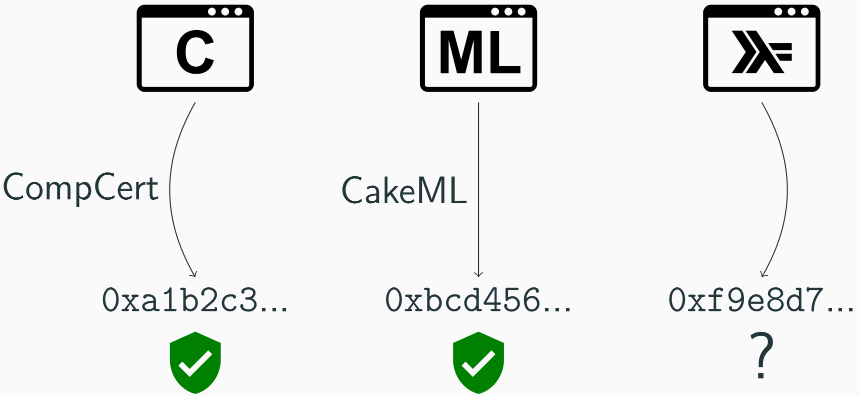

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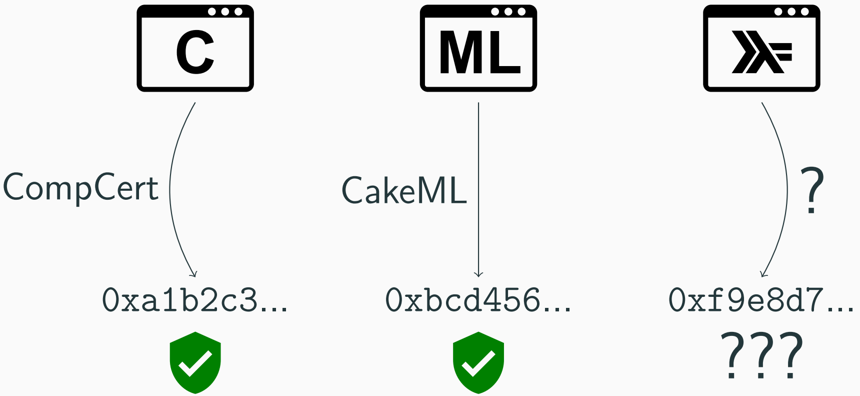

Our work

The **PureCake** project:

Our work

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a HOL4-**verified compiler** for

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The **PureCake** project:

a HOL4-**verified compiler** for

a **lazy**, **purely functional** language which

is inspired by **Haskell** and

targets **CakeML**

CakeML = a verified implementation of a subset of ML [POPL14]

Key contributions

Highlights

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- monadic reflection (monadic $\rightarrow$ imperative)
- CakeML as a back end for end-to-end verified compilation

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- sound equational reasoning
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***Not mechanically verified before**

This talk

Global overview + demands analysis

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For more details:

- Read our paper: cakeml.org/pldi23-purecake.pdf
- Visit our GitHub: github.com/cakeml/pure

Roadmap

Introduction

Source language

Compiler front end

Compiler back end

Connection with CakeML

A realistic functional language

PureLang has standard functional idioms ...

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```
fact :: Integer -> Integer -> Integer
fact a n =
  if n < 2 then a
  else fact (a * n) (n - 1)
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general recursion

A realistic functional language

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```
map :: (a -> b) -> [a] -> [b]
map f l = case l of
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          h:t -> f h : map f t
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algebraic data types + pattern-matching

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algebraic data types + pattern-matching

```
factorials :: [Integer]
factorials = map (fact 1) (numbers 0)
```

higher-order functions

A realistic subset of Haskell

... and Haskell extras

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```
numbers :: Integer -> [Integer]
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laziness $\rightarrow$ infinite data

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numbers :: Integer -> [Integer]
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```
main :: IO ()
main = do
  n <- readInt -- read from stdin
  let facts = take n factorials
  app (\i -> print $ toString i) facts
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laziness → infinite data

pure by default, monads for:

- sequencing
- stateful computations
- I/O

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Single `IO` monad for arrays, exceptions, and I/O (via FFI calls)

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pure by default, monads for:

- sequencing
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Single `IO` monad for arrays, exceptions, and I/O (via FFI calls)

Also: indentation-sensitivity, `do` notation, mutual recursion, …

Formal syntax

A tale of two ASTs... separate implementation and verification

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ce	e
compiler *expressions*	***semantic*** *expressions*

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- higher-level
- used in implementation
- includes `case`

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***compiler** expressions*

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***semantic** expressions*

- ground truth for semantics
- constructor operations: test name/arity equality & argument projection

Semantics — definitions

Operational semantics in layers:

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1. Weak-head evaluation: call-by-name, functional big-step

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2. Lift to unclocked evaluation

$$\mathsf{eval}_{\mathsf{wh}}\ e \stackrel{\mathsf{def}}{=} \begin{cases} wh & \text{for } \textbf{some } n,\ \ \mathsf{eval}_{\mathsf{wh}}^{n}\ e = wh \\ \mathsf{Timeout} & \text{for } \textbf{all } n,\ \ \mathsf{eval}_{\mathsf{wh}}^{n}\ e = \mathsf{Timeout} \end{cases}$$

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$$(\!|\, -,\ -,\ - \,|\!) : wh \to \kappa \to \sigma \to \text{itree}\ E\ A\ R$$

Finally, $[\![\, e \,]\!] \stackrel{\text{def}}{=} (\!|\, \text{eval}_{\text{wh}}\ e,\ \varepsilon,\ \varnothing \,|\!)$

Mechanised equational reasoning

Mechanise untyped applicative bisimulation, $\cong$ *[Abramsky, 1990]*

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Type system

Standard Hindley-Milner rules

Read the paper for more details

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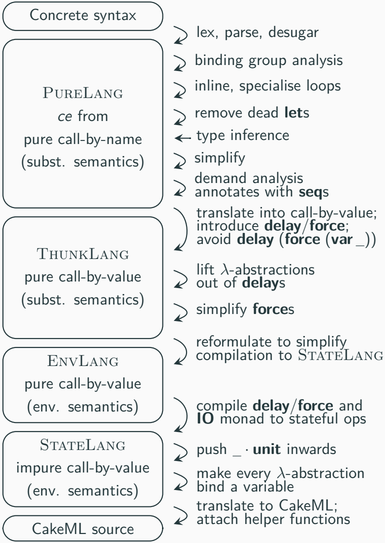
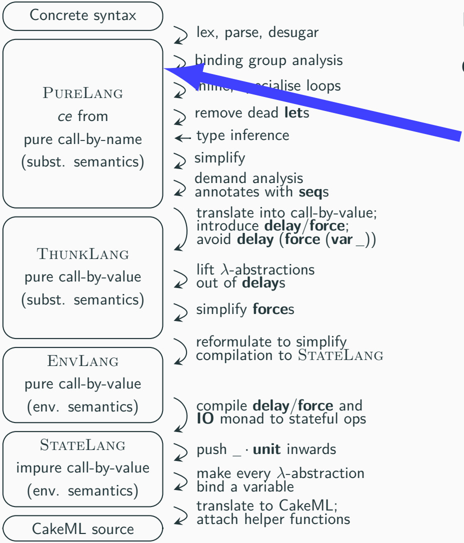
Compiler structure



Upcoming slides examine the compiler top to bottom

Compiler structure



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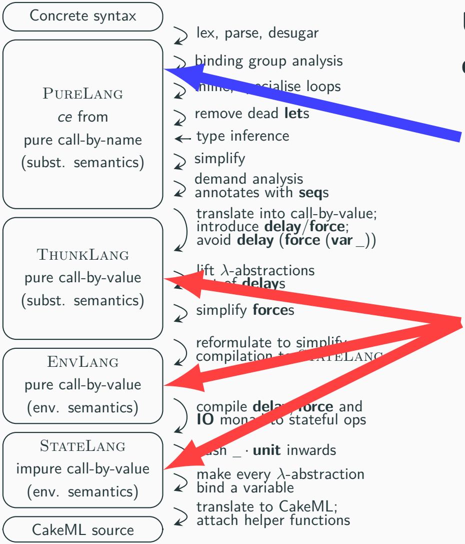
Frontend accepts PureLang



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Three intermediate languages, each with a specific focus

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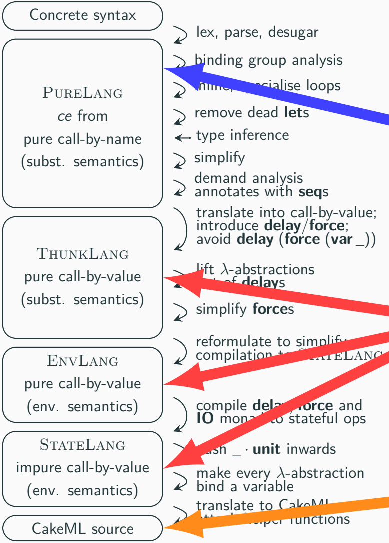

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Targets CakeML

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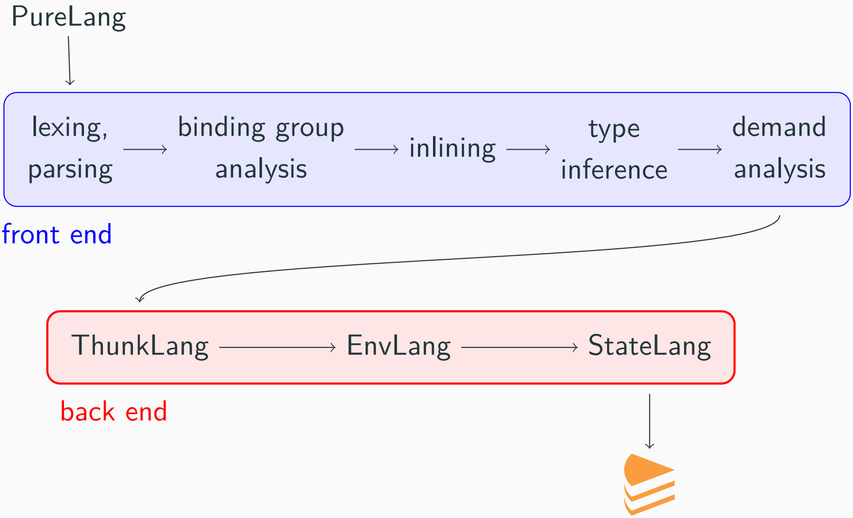

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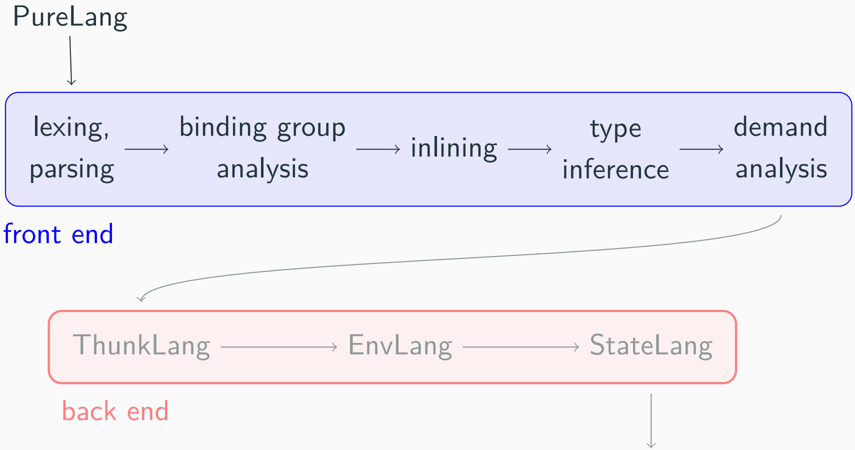

Parsing

Indentation-sensitive parsing expression grammar (PEG)

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- Symbolic sets of possible relations for each non-terminal
- Verified to terminate on all inputs

Binding group analysis

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z = 42
y = x + 1
x = w + y
w = 0
main
```

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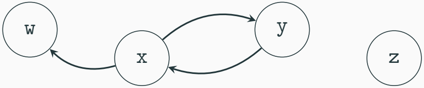

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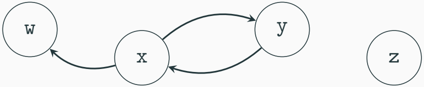
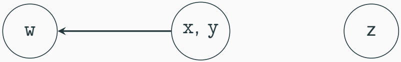


Pseudo-topological sort



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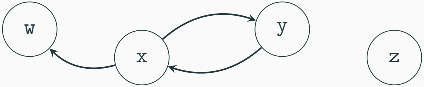

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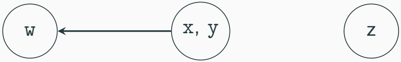

Transform code + tidy

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let w = 0 in
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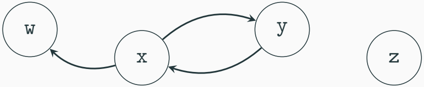

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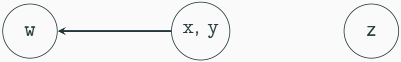

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Verified entirely within equational theory

Methodology — implementation vs. verification

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Prior work: (such as CakeML)

- Define implementation function: $\text{transform} : e \rightarrow e$
- Verify: $\text{wf}\ e \implies [\![\, \text{transform}\ e \,]\!] = [\![\, e \,]\!]$

Methodology — implementation vs. verification

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$$e \; \mathcal{R} \; e'$$

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$$\text{compile } ce = ce'$$

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- express core transformations
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syntactic relations

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$$\text{compile } ce = ce'$$

code transformation

- must fit in relation envelope: $ce \ \mathcal{R}' \ (\text{compile } ce)$ for **all** ce
- must satisfy bookkeeping

Methodology — workflow

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1. **Define** and **verify** $\mathcal{R}$: $e \mathrel{\mathcal{R}} e' \implies [\![e]\!] = [\![e']\!]$
   Three simulation proofs: one per layer of the semantics

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2. **Define** compile : $ce \to ce$
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4. **Compose** theorems:
   $$\text{wf } ce \implies [\![\text{desugar } ce]\!] = [\![\text{desugar} (\text{compile } ce)]\!]$$

Methodology — workflow

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   Three simulation proofs: one per layer of the semantics
2. **Define** compile : $ce \rightarrow ce$
3. **Verify** wf $ce \implies$ (desugar ce) $\mathcal{R}$ (desugar (compile ce))
4. **Compose** theorems:
   wf $ce \implies [\![\, \text{desugar } ce \,]\!] = [\![\, \text{desugar (compile } ce) \,]\!]$
5. **Integrate** into compiler, discharge wf ce

Methodology — workflow

1. **Define** and **verify** $\mathcal{R}$: $\quad e\ \mathcal{R}\ e' \implies [\![\, e \,]\!] = [\![\, e' \,]\!]$
   Three simulation proofs: one per layer of the semantics
2. **Define** compile : $ce \to ce$
3. **Verify** wf $ce \implies (\text{desugar}\ ce)\ \ \mathcal{R}\ \ (\text{desugar}\,(\text{compile}\ ce))$
4. **Compose** theorems:
   $$\text{wf}\ ce \implies [\![\, \text{desugar}\ ce \,]\!] = [\![\, \text{desugar}\,(\text{compile}\ ce) \,]\!]$$
5. **Integrate** into compiler, discharge wf ce

Separation of concerns for modularity and ease-of-verification

Demand analysis

Avoid excessive thunks — `acc` heap-allocated each recursive call!

```
fact acc n =          if n = 0
                      then acc
                      else fact (acc * n) (n - 1)
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- Prefix code with `seq`, including in recursive functions

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if (Seq v x) then y else z ≅ Seq v (if x then y else z)
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We have two distinct values for $\bot$: `Err` and `Div`

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- Fixpoint analysis for recursive functions

Roadmap

Introduction

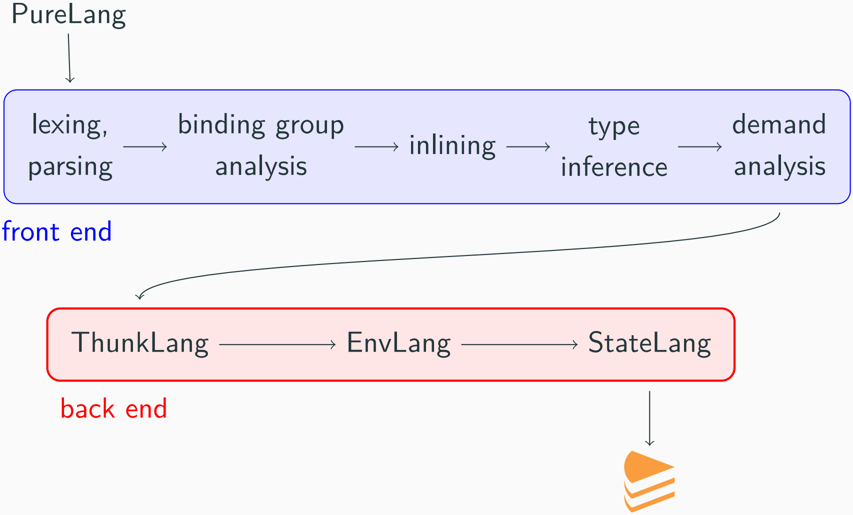
Source language

Compiler front end

Compiler back end

Connection with CakeML

Compiler structure



Compiler structure

PureLang

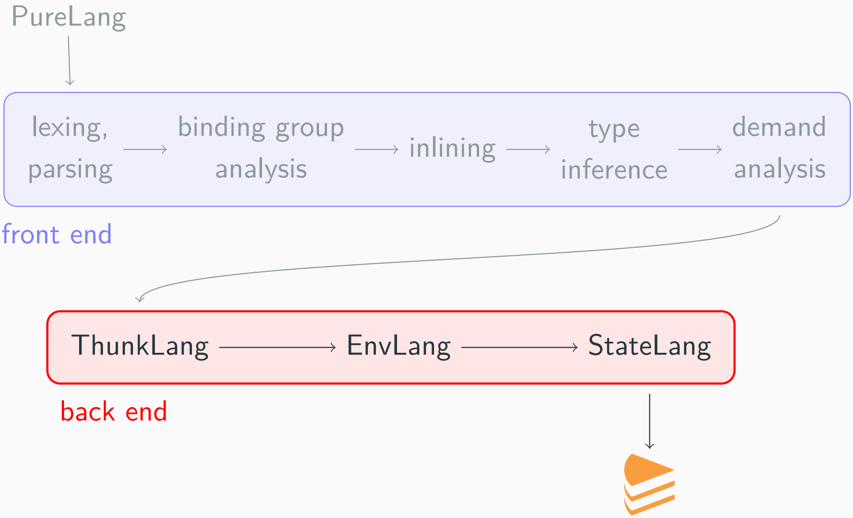
lexing, parsing → binding group analysis → inlining → type inference → demand analysis

front end

ThunkLang → EnvLang → StateLang

back end

Compiler structure



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Call-by-value semantics

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Verification: *seven* syntactic relations in total

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fact = Delay (\acc  n. Force n $ Force acc $
      if Force n = 0
          then Force acc
          else (Force fact) (Delay (Force acc * Force n))
                            (Delay (Force n - 1)))
```

EnvLang

Environment-based semantics + minor reformulations

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Semantics: ~~substitutions~~ **environments**

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Verification: focuses on the change in semantic style

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IO monad compiled to **stateful and I/O primitives, thunks shared statefully**

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[**true**, v] or [**false**, $\lambda\_.\ e$]

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Optimisation: simplify $\lambda\_.\ e$ and **unit**

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Reconciling differing semantic styles

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$$o_1 \xrightarrow{\Delta(o_1)} o_2 \xrightarrow{\Delta(o_2)} \cdots$$

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Oracles vs. ITrees

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$$\text{Vis}\ o\ k \begin{cases} \xrightarrow{k(a_1)} \text{Vis}\ o_1\ k_1\ \dots \\ \longrightarrow \text{Vis}\ o_2\ k_2\ \dots \\ \vdots \end{cases}$$

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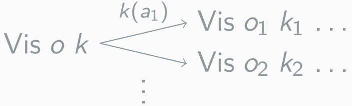

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$$\frac{\text{cakeml}\ e = \text{Some}\ code}{}$$

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$$\mathit{code} \textbf{ in memory of } \mathit{machine}$$

End-to-end correctness

purecake *str* = Some *ast*

cakeml *ast* = Some *code*

code **in memory of** *machine*

---

exists *ce* **such that**

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$$
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a **verified compiler** for a Haskell-like language

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Questions?

Backup slides

Future work

Only a first version! Many possible extensions, for example:

- Increasing source **expressivity** (e.g. for `case`)
- More Haskell 98 types, e.g. **typeclasses**
- More effective **demand analysis**
- Back end **optimisations**

A **verified REPL** for PureCake *[Sewell et. al., PLDI23]*

Evaluation — setup

Measure **execution time** and **memory allocations**

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- Five benchmarks, each accepting integer n input
  - primes: nth prime calculation
  - collatz: longest Collatz sequence for a number less than n
  - life: Conway's Game of Life for n generations
  - queens: solutions for the n-queens problem
  - qsort: imperative quicksort for an array of length n

Alternative compiler correctness

$$\frac{\text{frontend } str = \text{Some } (ce, \_)}{}$$

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ce **is type safe**

exists *ast* **such that**

Alternative compiler correctness

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