Retrofitting OCaml modules

An F^ω-inspired approach for a modern module system

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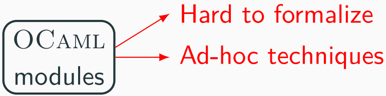
The big picture

OCaml
modules

The big picture

OCaml modules → Hard to formalize

The big picture



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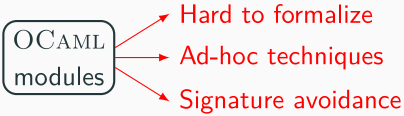

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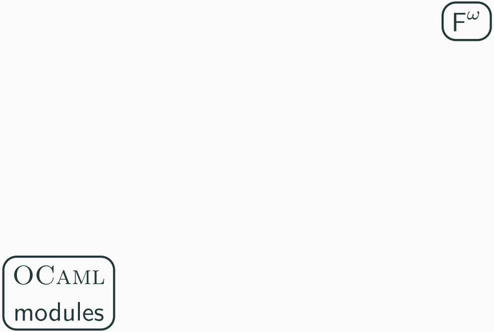

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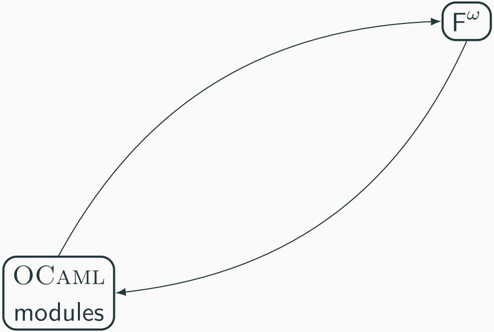


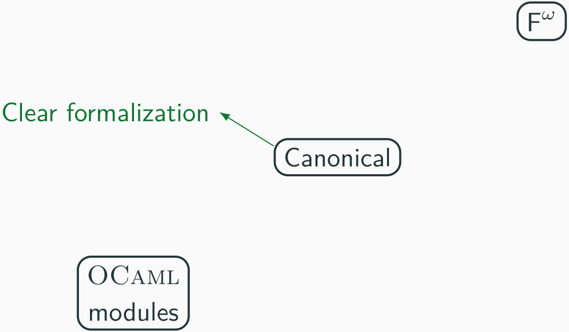
The big picture

F^ω

Canonical

OCaml modules

The big picture



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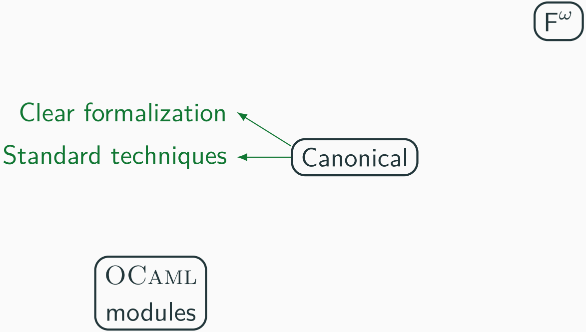

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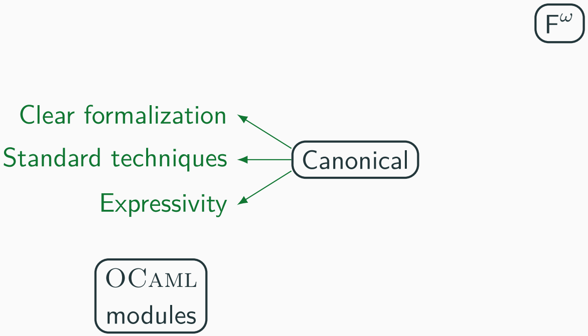

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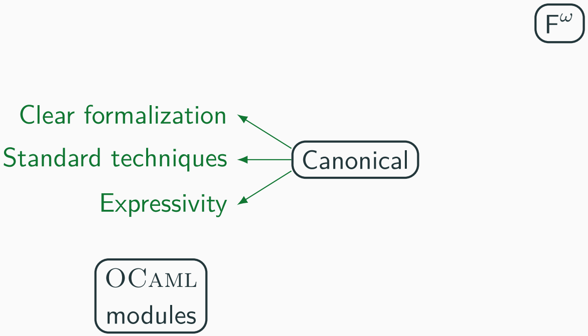

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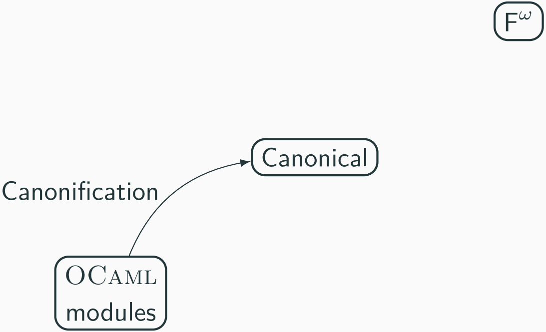

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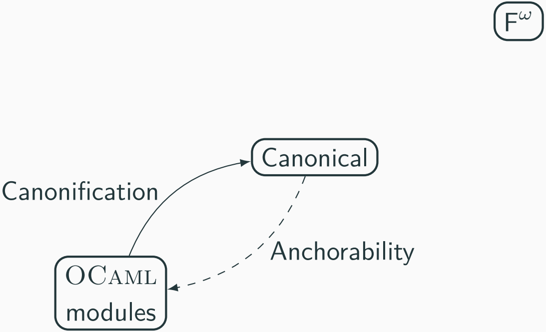

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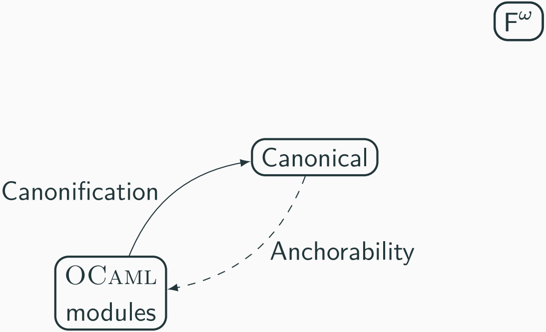

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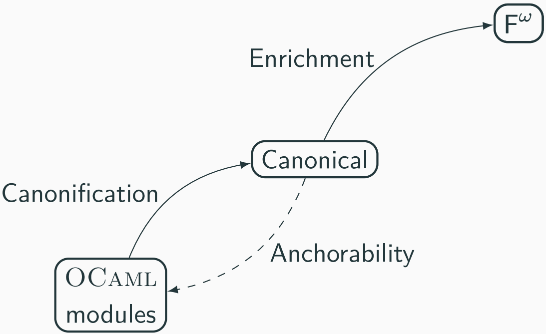

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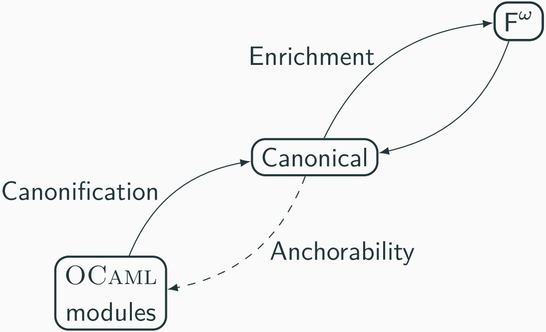

The OCaml Module system

As a module developer

As a module user

As a module developer

```
module Complex  = struct






end
```

As a module user

Basic modularity: modules, signatures and abstraction

As a module developer

```
module Complex  = struct
  type t = float * float




end
```

As a module user

Basic modularity: modules, signatures and abstraction

As a module developer

```
module Complex  = struct
  type t = float * float
  let zero = (0., 0.)
  let one = (1., 0.)
  let add = ...
  let mult = ...
end
```

As a module user

Basic modularity: modules, signatures and abstraction

As a module developer

```
module Complex  = struct
  type t = float * float
  let zero = (0., 0.)
  let one = (1., 0.)
  let add = ...
  let mult = ...
end

module type Ring = sig





end
```

As a module user

Basic modularity: modules, signatures and abstraction

As a module developer

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module Complex  = struct
  type t = float * float
  let zero = (0., 0.)
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  let mult = ...
end

module type Ring = sig
  type t




end
```

As a module user

Basic modularity: modules, signatures and abstraction

As a module developer

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module Complex  = struct
  type t = float * float
  let zero = (0., 0.)
  let one = (1., 0.)
  let add = ...
  let mult = ...
end

module type Ring = sig
  type t
  val zero : t
  val one : t
  val add : t -> t -> t
  val mult : t -> t -> t
end
```

As a module user

As a module developer

```
module Complex : Ring = struct
  type t = float * float
  let zero = (0., 0.)
  let one = (1., 0.)
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module type Ring = sig
  type t
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module type Ring = sig
  type t
  val zero : t
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As a module user

```
module Polynomials =






end
```

Basic modularity: modules, signatures and abstraction

As a module developer

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module Complex : Ring = struct
  type t = float * float
  let zero = (0., 0.)
  let one = (1., 0.)
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module type Ring = sig
  type t
  val zero : t
  val one : t
  val add : t -> t -> t
  val mult : t -> t -> t
end
```

As a module user

```
module Polynomials =
  functor (R: Ring) -> struct






end
```

```





```

Basic modularity: modules, signatures and abstraction

As a module developer

```
module Complex : Ring = struct
  type t = float * float
  let zero = (0., 0.)
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end
```

As a module user

```
module Polynomials =
  functor (R: Ring) -> struct
    type t = R.t list
    let zero = []
    let one = [R.one]
    let add = ...
    let mult = ...
end
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Basic modularity: modules, signatures and abstraction

As a module developer

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module Complex : Ring = struct
  type t = float * float
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  functor (R: Ring) -> struct
    type t = R.t list
    let zero = []
    let one = [R.one]
    let add = ...
    let mult = ...
end
```

```
module CX =
```

Basic modularity: modules, signatures and abstraction

As a module developer

```
module Complex : Ring = struct
  type t = float * float
  let zero = (0., 0.)
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  let add = ...
  let mult = ...
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module type Ring = sig
  type t
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As a module user

```
module Polynomials =
  functor (R: Ring) -> struct
    type t = R.t list
    let zero = []
    let one = [R.one]
    let add = ...
    let mult = ...
end
```

```
module CX =
  Polynomials(Complex)
```

Basic modularity: modules, signatures and abstraction

As a module developer

```
module Complex : Ring = struct
  type t = float * float
  let zero = (0., 0.)
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module type Ring = sig
  type t
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end
```

As a module user

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module Polynomials =
  functor (R: Ring) -> struct
    type t = R.t list
    let zero = []
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    let add = ...
    let mult = ...
end
```

```
module CX =
  Polynomials(Complex)

module CXY =

```

Basic modularity: modules, signatures and abstraction

As a module developer

```
module Complex : Ring = struct
  type t = float * float
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As a module user

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module Polynomials =
  functor (R: Ring) -> struct
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end
```

```
module CX =
  Polynomials(Complex)

module CXY =
  Polynomials(Polynomials(Complex))
```

What flavor for you functor ?

What flavor for you functor ?

Generative

What flavor for you functor ?

Generative

Applicative

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Generative

Functors as parameterized *sub-programs*

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Functors as parameterized *sub-programs*

- Internal state / effects

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Functors as parameterized *sub-programs*

- Internal state / effects
- Dynamic choice of implementation
  (via 1^{st} class modules)

Applicative

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Functors as parameterized *sub-programs*

- Internal state / effects
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```
module SymbolTable () =
```

Applicative

What flavor for you functor ?

Generative

Functors as parameterized *sub-programs*

- Internal state / effects
- Dynamic choice of implementation (via 1^{st} class modules)

```
module SymbolTable () = struct
  type t = int
  let x = ref 0
  ...
end
```

Applicative

What flavor for you functor ?

Generative

Functors as parameterized *sub-programs*

- Internal state / effects
- Dynamic choice of implementation (via 1^{st} class modules)

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module SymbolTable () = (struct
  type t = int
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Functors as parameterized *sub-programs*

- Internal state / effects
- Dynamic choice of implementation (via 1st class modules)

```
module SymbolTable () = (struct
  type t = int
  let x = ref 0
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end : sig type t ... end)

module ST1 = SymboleTable()
module ST2 = SymboleTable()
```

Applicative

What flavor for you functor ?

Generative

Functors as parameterized *sub-programs*

- Internal state / effects
- Dynamic choice of implementation (via 1st class modules)

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module SymbolTable () = (struct
  type t = int
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module ST1 = SymboleTable()
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(* ST1.t ≠ ST2.t *) ×
```

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Functors as parameterized *libraries*

What flavor for you functor ?

Generative

Functors as parameterized *sub-programs*

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Applicative

Functors as parameterized *libraries*

- Purity
- Static choice of implementation

```
module Set (E:OrderedType) = struct
```

What flavor for you functor ?

Generative

Functors as parameterized *sub-programs*

- Internal state / effects
- Dynamic choice of implementation (via 1st class modules)

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Functors as parameterized *libraries*

- Purity
- Static choice of implementation

```
module Set (E:OrderedType) = struct
  type t = E.t list
  let empty : t = []
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What flavor for you functor ?

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Functors as parameterized *sub-programs*

- Internal state / effects
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Functors as parameterized *libraries*

- Purity
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```
module Set (E:OrderedType) = (struct
  type t = E.t list
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module S1 = Set(Integer)
module S2 = Set(Integer)
```

What flavor for you functor ?

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Functors as parameterized *sub-programs*

- Internal state / effects
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module S1 = Set(Integer)
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(* S1.t = S2.t *) ✓
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What flavor for you functor ?

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Functors as parameterized *sub-programs*

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Functors as parameterized *libraries*

- Purity
- Static choice of implementation

→ *same applications* produce same results

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Functors as parameterized *sub-programs*

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(* S1.t = S2.t *) ✓
```

Applicativity granularity

```
module X1 = struct
  type t = int
  ...
end

module X2 = struct
  type t = int
  ...
end

Set(X1).t =? Set(X2).t
```

Applicativity granularity

Types only

```
module X1 = struct
  type t = int
  ...
end

module X2 = struct
  type t = int
  ...
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Set(X1).t =? Set(X2).t
```

Applicativity granularity

Types only

- Sound: types only depend on types

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module X1 = struct
  type t = int
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Set(X1).t =? Set(X2).t
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Applicativity granularity

Types only

- Sound: types only depend on types
  $\rightarrow$ assumes the functor's body only depends on types fields

```
module X1 = struct
  type t = int
  ...
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  type t = int
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Set(X1).t =? Set(X2).t
```

Applicativity granularity

Types only

- Sound: types only depend on types
  $\rightarrow$ assumes the functor's body only depends on types fields

```
module X1 = struct
  type t = int
  let compare = (<)
end

module X2 = struct
  type t = int
  let compare = (>)
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Set(X1).t =? Set(X2).t
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Applicativity granularity

Types only

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Types and values

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Applicativity granularity

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- Sound: types only depend on types
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Types and values

- *Abstraction safe*: dynamically equivalent

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Types only

- Sound: types only depend on types
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Types and values

- *Abstraction safe*: dynamically equivalent
  $\rightarrow$ tracking equality of values

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Applicativity granularity

Types only

- Sound: types only depend on types
  $\rightarrow$ assumes the functor's body only depends on types fields

Types and values - *fine grained*

- *Abstraction safe*: dynamically equivalent
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module X1 = struct
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Types and values - *fine grained*

- *Abstraction safe*: dynamically equivalent
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Modules - *coarse grained*

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module X1 = struct
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Set(X1).t =? Set(X2).t
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Types and values - *fine grained*

- *Abstraction safe*: dynamically equivalent
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Modules - *coarse grained*

- Syntactic criterion

```
module X1 = struct
  type t = int
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Set(X1).t =? Set(X2).t
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Applicativity granularity

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Types and values - *fine grained*

- *Abstraction safe*: dynamically equivalent
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Modules - *coarse grained*

- Syntactic criterion
  → tracking of module *aliasing*

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module X1 = struct
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Modules - *coarse grained*

- Syntactic criterion
  $\rightarrow$ tracking of module *aliasing*

```
module X1 = struct
  type t = int
  let compare = (<)
end

module X2 = X1




Set(X1).t =? Set(X2).t
```

Applicativity granularity

Types only

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  → assumes the functor's body only depends on types fields

Types and values - *fine grained*

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Modules - *coarse grained*

- Syntactic criterion
  → tracking of module *aliasing*

```
module X1 = struct
  type t = int
  let compare = (<)
end

module X2 = X1




Set(X1).t =? Set(X2).t
```

Aliases and transparent ascription

Modules

Signatures

Aliases and transparent ascription

Modules

```
module X1 = struct ... end
```

Signatures

Aliases and transparent ascription

Modules

```
module X1 = struct ... end
```

Signatures

```
module X1 : sig ... end
```

Aliases and transparent ascription

- Same module

Modules

```
module X1 = struct ... end
```

Signatures

```
module X1 : sig ... end
```

Aliases and transparent ascription

- Same module

Modules

```
module X1 = struct ... end

module X2 = X1
```

Signatures

```
module X1 : sig ... end
```

Aliases and transparent ascription

- Same module

Modules

```
module X1 = struct ... end

module X2 = X1
```

Signatures

```
module X1 : sig ... end

module X2 : (= X1)
```

Aliases and transparent ascription

- Same module
- Subtyping (not code-free)

Modules

```
module X1 = struct ... end

module X2 = X1
```

Signatures

```
module X1 : sig ... end

module X2 : (= X1)
```

Aliases and transparent ascription

- Same module
- Subtyping (not code-free)

Modules

```
module X1 = struct ... end

module X2 = X1

module X3 = (X1 : S)
```

Signatures

```
module X1 : sig ... end

module X2 : (= X1)
```

Aliases and transparent ascription

- Same module
- Subtyping (not code-free)

Modules

```
module X1 = struct ... end

module X2 = X1

module X3 = (X1 : S)

module X4 = (X1 < S)
```

Signatures

```
module X1 : sig ... end

module X2 : (= X1)
```

Aliases and transparent ascription

- Same module
- Subtyping (not code-free)

Modules

```
module X1 = struct ... end

module X2 = X1

module X3 = (X1 : S)

module X4 = (X1 < S)
```

Signatures

```
module X1 : sig ... end

module X2 : (= X1)

module X3 : S
```

Aliases and transparent ascription

- Same module
- Subtyping (not code-free)

Modules

```
module X1 = struct ... end

module X2 = X1

module X3 = (X1 : S)

module X4 = (X1 < S)


```

Signatures

```
module X1 : sig ... end

module X2 : (= X1)

module X3 : S

module X4 : (= X1 < S)


```

Aliases and transparent ascription

- Same module
- Subtyping (not code-free) $\rightarrow$ transparent/opaque

Modules

```
module X1 = struct ... end

module X2 = X1

module X3 = (X1 : S)

module X4 = (X1 < S)
```

Signatures

```
module X1 : sig ... end

module X2 : (= X1)

module X3 : S

module X4 : (= X1 < S)
```

Aliases and transparent ascription

- Same module
- Subtyping (not code-free) $\rightarrow$ transparent/opaque
- Functor applications

Modules

```
module X1 = struct ... end

module X2 = X1

module X3 = (X1 : S)

module X4 = (X1 < S)


```

Signatures

```
module X1 : sig ... end

module X2 : (= X1)

module X3 : S

module X4 : (= X1 < S)


```

Aliases and transparent ascription

- Same module
- Subtyping (not code-free) $\rightarrow$ transparent/opaque
- Functor applications

Modules

```
module X1 = struct ... end

module X2 = X1

module X3 = (X1 : S)

module X4 = (X1 < S)

module X5 = (functor (Y:S) -> Y)(X1)
```

Signatures

```
module X1 : sig ... end

module X2 : (= X1)

module X3 : S

module X4 : (= X1 < S)
```

Aliases and transparent ascription

- Same module
- Subtyping (not code-free) $\rightarrow$ transparent/opaque
- Functor applications

Modules

```
module X1 = struct ... end

module X2 = X1

module X3 = (X1 : S)

module X4 = (X1 < S)

module X5 = (functor (Y:S) -> Y)(X1)
```

Signatures

```
module X1 : sig ... end

module X2 : (= X1)

module X3 : S

module X4 : (= X1 < S)

module X5 : (= X1 < S)
```

The canonical system

Canonical signatures - canonification

Enriched syntax

→ F^{ω} quantifiers

Key mechanisms

module M

- module X
  - type t
  - val $x : t$
  - type $u = \text{int} \times t$
- module F (Y : sig type t end)
  - type t
  - type $u = Y.t$
- module G ()
  - type t
  - val $x : t$
- module type T
  - type t
  - val $x : t$

Canonical signatures - canonification

Enriched syntax

→ F^ω quantifiers

Key mechanisms

module M
- module X
  - type t
  - val x : t
  - type u = int × t
- module F (Y : sig type t end)
  - type t
  - type u = $Y.t$
- module G ()
  - type t
  - val x : t
- module type T
  - type t
  - val x : t

Canonical signatures - canonification

Enriched syntax

$\rightarrow$ F^{ω} quantifiers

- Existential for ascription

Key mechanisms

module M

- module X
  - $\exists\alpha$
    - type $t = \alpha$
    - val $x : t$
    - type $u = \text{int} \times t$
- module F (Y : sig type t end)
  - type t
  - type $u = Y.t$
- module G ()
  - type t
  - val $x : t$
- module type T
  - type t
  - val $x : t$

Canonical signatures - canonification

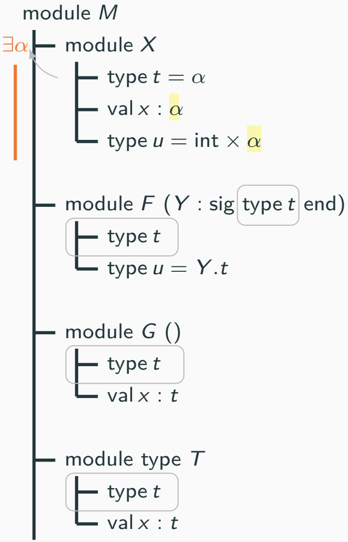
Enriched syntax

→ F^ω quantifiers

- Existential for ascription

Key mechanisms

- Existential lifting



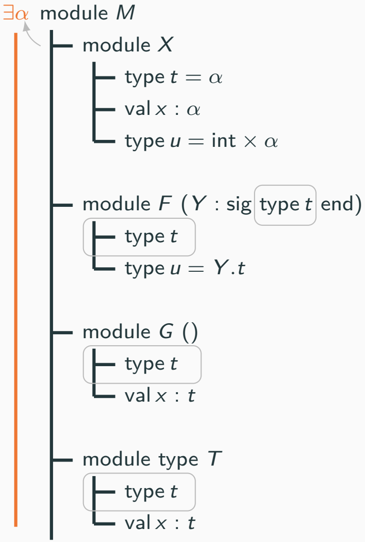
Enriched syntax

→ F^ω quantifiers

- Existential for ascription
- Universal for functors

Key mechanisms

- Existential lifting



Canonical signatures - canonification

Enriched syntax

$\rightarrow$ F^{ω} quantifiers

- Existential for ascription
- Universal for functors

Key mechanisms

- Existential lifting

$\exists\alpha$ module M

- module X
  - type $t = \alpha$
  - val $x : \alpha$
  - type $u = \text{int} \times \alpha$
- module $F : \forall\beta.(Y : \text{sig type } t = \beta \text{ end})$
  - type t
  - type $u = \beta$
- module G ()
  - type t
  - val $x : t$
- module type T
  - type t
  - val $x : t$

Canonical signatures - canonification

Enriched syntax

$\rightarrow$ F^{ω} quantifiers

- Existential for ascription
- Universal for functors

Key mechanisms

- Existential lifting

$\exists\alpha$ module M

- module X
  - type $t = \alpha$
  - val $x : \alpha$
  - type $u = \text{int} \times \alpha$
- module $F : \forall\beta.(Y : \text{sig type } t = \beta \text{ end})$
  - $\exists\gamma$ type $t = \gamma$
  - type $u = \beta$
- module G ()
  - type t
  - val $x : t$
- module type T
  - type t
  - val $x : t$

Canonical signatures - canonification

Enriched syntax

$\rightarrow$ F^{ω} quantifiers

- Existential for ascription
- Universal for functors

Key mechanisms

- Existential lifting
- Skolemization

```
∃α  module M
├─ module X
│  ├─ type t = α
│  ├─ val x : α
│  └─ type u = int × α
│
∃γ' ├─ module F : ∀β.(Y : sig type t = β end)
│  ├─ type t = γ'(β)
│  └─ type u = β
│
├─ module G ()
│  ├─ type t
│  └─ val x : t
│
└─ module type T
   ├─ type t
   └─ val x : t
```

Canonical signatures - canonification

Enriched syntax

$\rightarrow$ F^ω quantifiers

- Existential for ascription
- Universal for functors

Key mechanisms

- Existential lifting
- Skolemization

$\exists \alpha, \gamma'$ module M

- module X
  - type $t = \alpha$
  - val $x : \alpha$
  - type $u = \text{int} \times \alpha$
- module $F : \forall \beta.(Y : \text{sig type } t = \beta \text{ end})$
  - type $t = \gamma'(\beta)$
  - type $u = \beta$
- module G ()
  - type t
  - val $x : t$
- module type T
  - type t
  - val $x : t$

Canonical signatures - canonification

Enriched syntax

$\rightarrow$ F^{ω} quantifiers

- Existential for ascription
- Universal for functors

Key mechanisms

- Existential lifting
- Skolemization

$\exists \alpha, \gamma'$ module M

- module X
  - type $t = \alpha$
  - val $x : \alpha$
  - type $u = \text{int} \times \alpha$
- module $F : \forall \beta.(Y : \text{sig type } t = \beta \text{ end})$
  - type $t = \gamma'(\beta)$
  - type $u = \beta$
- module G ()
  - $\exists \delta$ type $t = \delta$
  - val $x : t$
- module type T
  - type t
  - val $x : t$

Canonical signatures - canonification

Enriched syntax

→ F^{ω} quantifiers

- Existential for ascription
- Universal for functors

Key mechanisms

- Existential lifting
- Skolemization

$\exists \alpha, \gamma'$ module M

- module X
  - type $t = \alpha$
  - val $x : \alpha$
  - type $u = \text{int} \times \alpha$
- module $F : \forall \beta.(Y : \text{sig type } t = \beta \text{ end})$
  - type $t = \gamma'(\beta)$
  - type $u = \beta$
- module G () : $\exists \delta$
  - type $t = \delta$
  - val $x : \delta$
- module type T
  - type t
  - val $x : t$

Canonical signatures - canonification

Enriched syntax

$\rightarrow$ F^ω quantifiers

- Existential for ascription
- Universal for functors
- Lambda for module types

Key mechanisms

- Existential lifting
- Skolemization

$\exists \alpha, \gamma'$ module M

- module X
  - type $t = \alpha$
  - val $x : \alpha$
  - type $u = \text{int} \times \alpha$
- module $F : \forall \beta.(Y : \text{sig type } t = \beta \text{ end})$
  - type $t = \gamma'(\beta)$
  - type $u = \beta$
- module $G\ () : \exists \delta$
  - type $t = \delta$
  - val $x : \delta$
- module type T
  - $\lambda \rho$
    - type $t = \rho$
    - val $x : t$

Canonical signatures - canonification

Enriched syntax

$\rightarrow$ F^{ω} quantifiers

- Existential for ascription
- Universal for functors
- Lambda for module types

Key mechanisms

- Existential lifting
- Skolemization

$\exists \alpha, \gamma'$ module M

- module X
  - type $t = \alpha$
  - val $x : \alpha$
  - type $u = \text{int} \times \alpha$
- module $F : \forall \beta.(Y : \text{sig type } t = \beta \text{ end})$
  - type $t = \gamma'(\beta)$
  - type $u = \beta$
- module $G\ () : \exists \delta$
  - type $t = \delta$
  - val $x : \delta$
- module type $T : \lambda \rho$
  - type $t = \rho$
  - val $x : \rho$

Module identities

module M

- module X
  - type $t = \alpha$
  - val $x : \alpha$
  - type $u = \text{int} \times \alpha$
- module $X' : (= X < S)$

Module identities

- Use type abstraction mechanisms to track module identities

module M
- module X
  - type $t = \alpha$
  - val $x : \alpha$
  - type $u = \text{int} \times \alpha$
- module $X' : (= X < S)$

Module identities

- Use type abstraction mechanisms to track module identities
- Add abstract id fields, with a special treatment by typing rules

module M

- module X
  - type $t = \alpha$
  - val $x : \alpha$
  - type $u = \text{int} \times \alpha$
- module $X' : (= X < S)$

Module identities

- Use type abstraction mechanisms to track module identities
- Add abstract `id` fields, with a special treatment by typing rules

$\exists \alpha_X$ module M

- module $X = \alpha_X$
  - type $t = \alpha$
  - val $x : \alpha$
  - type $u = \text{int} \times \alpha$
- module $X' : (= X < S)$

Module identities

- Use type abstraction mechanisms to track module identities
- Add abstract `id` fields, with a special treatment by typing rules

$\exists\alpha_X$ module M

- module $X = \alpha_X$
  - type $t = \alpha$
  - val $x : \alpha$
  - type $u = \text{int} \times \alpha$
- module $X' = \alpha_X$

Module identities

- Use type abstraction mechanisms to track module identities
- Add abstract `id` fields, with a special treatment by typing rules

$\exists \alpha_X$ module M

- module $X = \alpha_X$
  - type $t = \alpha$
  - val $x : \alpha$
  - type $u = \text{int} \times \alpha$
- module $X' = \alpha_X$
  - type $t = \alpha$
  - val $x : \alpha$
  - ~~type $u = \text{int} \times \alpha$~~

Module identities

- Use type abstraction mechanisms to track module identities
- Add abstract `id` fields, with a special treatment by typing rules

$\exists \alpha_X$ module M

- module $X = \alpha_X$
  - type $t = \alpha$
  - val $x : \alpha$
  - type $u = \text{int} \times \alpha$
- module $X' = \alpha_X$
  - type $t = \alpha$
  - val $x : \alpha$
  - ~~type $u = \text{int} \times \alpha$~~

Canonical grammar

Invariants

Canonical grammar

Invariants

- Quantifier positions ($\exists$,$\forall$,λ)

Canonical grammar

Invariants

- Quantifier positions ($\exists$,$\forall$,λ)
- Identities
  $\rightarrow$ when *reachable by a path*

Canonical grammar

Abstract signatures

$\mathcal{S} ::= \exists\overline{\alpha}.\mathcal{C}$

Invariants

- Quantifier positions ($\exists$,$\forall$,λ)
- Identities
  $\rightarrow$ when *reachable by a path*

Canonical grammar

Abstract signatures

$$\mathcal{S} ::= \exists\overline{\alpha}.\mathcal{C}$$

Identity signatures

$$\mathcal{C} ::= (\tau, \mathcal{R})$$

Invariants

- Quantifier positions ($\exists$,$\forall$,λ)
- Identities
  $\rightarrow$ when *reachable by a path*

Canonical grammar

Abstract signatures

$$\mathcal{S} ::= \exists\overline{\alpha}.\mathcal{C}$$

Identity signatures

$$\mathcal{C} ::= (\tau, \mathcal{R})$$

Invariants

- Quantifier positions ($\exists$,$\forall$,λ)
- Identities
  $\rightarrow$ when *reachable by a path*

Types

$$\tau ::= \alpha \mid \alpha(\overline{\tau})$$

Canonical grammar

Invariants

- Quantifier positions ($\exists$,$\forall$,λ)
- Identities
  $\rightarrow$ when *reachable by a path*

Abstract signatures

$$\mathcal{S} ::= \exists\overline{\alpha}.\mathcal{C}$$

Identity signatures

$$\mathcal{C} ::= (\tau, \mathcal{R})$$

Manifest signatures

$$\begin{aligned} \mathcal{R} ::= & \ \texttt{sig}\ \overline{\mathcal{D}}\ \texttt{end} && \textit{Structural signature} \\ | & \ \forall\overline{\alpha}.\,\mathcal{C} \rightarrow \mathcal{R} && \textit{Applicative functor} \\ | & \ () \rightarrow \exists\overline{\alpha}.\mathcal{R} \end{aligned}$$

Types

$$\begin{aligned} \tau ::= & \ \alpha \\ | & \ \alpha(\overline{\tau}) \end{aligned}$$

Canonical grammar

Invariants

- Quantifier positions ($\exists$,$\forall$,λ)
- Identities
  $\rightarrow$ when *reachable by a path*

Abstract signatures

$$\mathcal{S} ::= \exists\overline{\alpha}.\mathcal{C}$$

Identity signatures

$$\mathcal{C} ::= (\tau, \mathcal{R})$$

Manifest signatures

$\mathcal{R} ::= \texttt{sig}\ \overline{\mathcal{D}}\ \texttt{end}$	*Structural signature*
$\mid \forall\overline{\alpha}.\,\mathcal{C} \rightarrow \mathcal{R}$	*Applicative functor*
$\mid () \rightarrow \exists\overline{\alpha}.\mathcal{R}$	*Generative functor*

Declarations

$$\begin{aligned}\mathcal{D} ::=\ & \mathsf{val}\, x : \tau \\ \mid\ & \mathsf{type}\, t = \tau \\ \mid\ & \mathsf{module}\, X : \mathcal{C} \\ \mid\ & \mathsf{module\ type}\ \ T = \lambda\overline{\alpha}.\mathcal{C}\end{aligned}$$

Types

$$\begin{aligned}\tau ::=\ & \alpha \\ \mid\ & \alpha(\overline{\tau})\end{aligned}$$

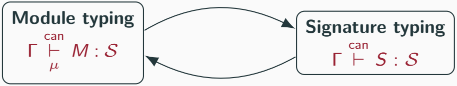
The canonical system

The canonical system

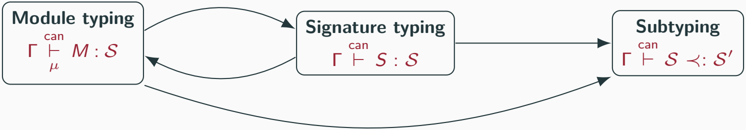
Signature typing

$\Gamma \overset{\text{can}}{\vdash} S : \mathcal{S}$

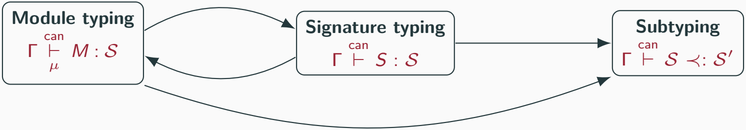
The canonical system



The canonical system



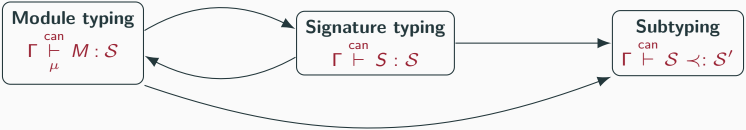
The canonical system



C-Typ-Mod-Struct

$$\frac{}{\Gamma \overset{\text{can}}{\underset{\mu}{\vdash}} \texttt{struct}\ \overline{B}\ \texttt{end} :}$$

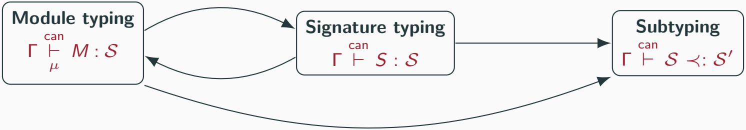
The canonical system



C-Typ-Mod-Struct

$$\frac{\Gamma \underset{\mu}{\overset{\text{can}}{\vdash}} \overline{B} : \exists \overline{\alpha}.\overline{\mathcal{D}}}{\Gamma \underset{\mu}{\overset{\text{can}}{\vdash}} \texttt{struct}\ \overline{B}\ \texttt{end} :}$$

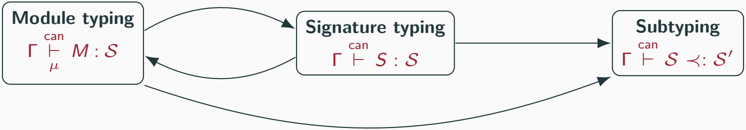
The canonical system



C-Typ-Mod-Struct

$$\frac{\Gamma \overset{\text{can}}{\underset{\mu}{\vdash}} \overline{B} : \exists \overline{\alpha}.\overline{\mathcal{D}}}{\Gamma \overset{\text{can}}{\underset{\mu}{\vdash}} \texttt{struct}\ \overline{B}\ \texttt{end} : \exists \overline{\alpha}\quad .\left(\quad , \texttt{sig}\ \overline{\mathcal{D}}\ \texttt{end}\right)}$$

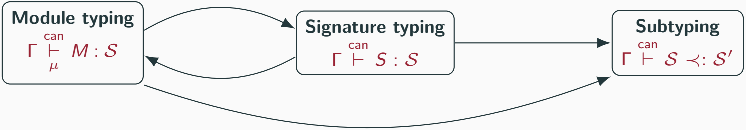
The canonical system



C-Typ-Mod-Struct

$$\frac{\Gamma \overset{\text{can}}{\underset{\mu}{\vdash}} \overline{B} : \exists \overline{\alpha}. \overline{\mathcal{D}}}{\Gamma \overset{\text{can}}{\underset{\mu}{\vdash}} \texttt{struct}\ \overline{B}\ \texttt{end} : \exists \overline{\alpha}, \alpha. \left(\quad , \texttt{sig}\ \overline{\mathcal{D}}\ \texttt{end} \right)}$$

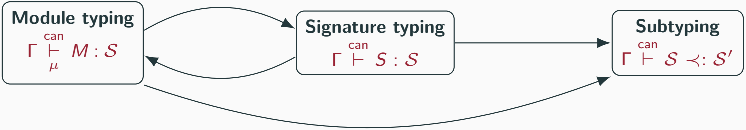
The canonical system



C-Typ-Mod-Struct

$$\frac{\Gamma \underset{\mu}{\overset{\text{can}}{\vdash}} \overline{B} : \exists \overline{\alpha}.\overline{\mathcal{D}}}{\Gamma \underset{\mu}{\overset{\text{can}}{\vdash}} \texttt{struct}\ \overline{B}\ \texttt{end} : \exists \overline{\alpha}, \alpha.\ \left(\alpha, \texttt{sig}\ \overline{\mathcal{D}}\ \texttt{end}\right)}$$

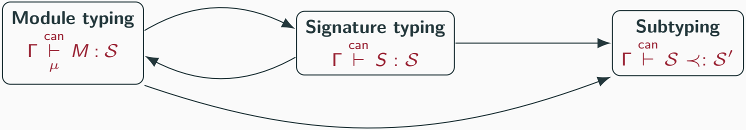
The canonical system



C-Typ-Mod-AppGen

$$\frac{}{\Gamma \overset{\text{can}}{\underset{\text{gen}}{\vdash}} P() :}$$

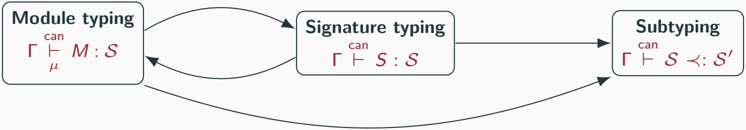
The canonical system



C-Typ-Mod-AppGen

$$\frac{\Gamma \underset{\mu}{\overset{\text{can}}{\vdash}} P : (\tau, () \to \exists \overline{\alpha}.\mathcal{R})}{\Gamma \underset{\text{gen}}{\overset{\text{can}}{\vdash}} P() :}$$

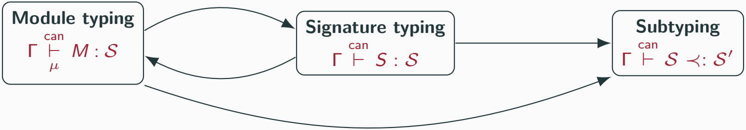
The canonical system



C-Typ-Mod-AppGen

$$\frac{\Gamma \overset{\text{can}}{\underset{\mu}{\vdash}} P : (\tau, () \to \exists\overline{\alpha}.\mathcal{R})}{\Gamma \overset{\text{can}}{\underset{\text{gen}}{\vdash}} P() : \exists\overline{\alpha} \quad .(\quad , \mathcal{R})}$$

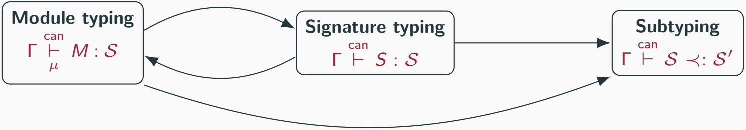
The canonical system



C-Typ-Mod-AppGen

$$\frac{\Gamma \overset{\text{can}}{\underset{\mu}{\vdash}} P : (\tau, () \rightarrow \exists \overline{\alpha}.\mathcal{R})}{\Gamma \overset{\text{can}}{\underset{\text{gen}}{\vdash}} P() : \exists \overline{\alpha}, \alpha.(\quad, \mathcal{R})}$$

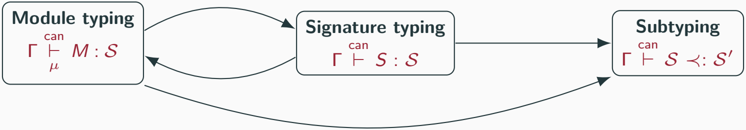
The canonical system



C-Typ-Mod-AppGen

$$\frac{\Gamma \overset{\text{can}}{\underset{\mu}{\vdash}} P : (\tau, () \to \exists \overline{\alpha}.\mathcal{R})}{\Gamma \overset{\text{can}}{\underset{\text{gen}}{\vdash}} P() : \exists \overline{\alpha}, \alpha. (\alpha, \mathcal{R})}$$

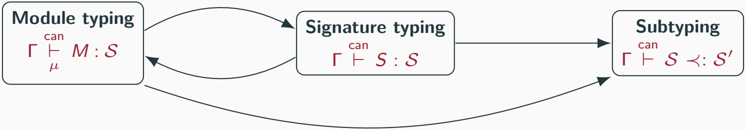
The canonical system



C-Typ-Mod-AppGen

$$\frac{\Gamma \overset{\text{can}}{\underset{\mu}{\vdash}} P : (\tau, () \rightarrow \exists \overline{\alpha}.\mathcal{R})}{\Gamma \overset{\text{can}}{\underset{\text{gen}}{\vdash}} P() : \exists \overline{\alpha}, \alpha. (\alpha, \mathcal{R})}$$

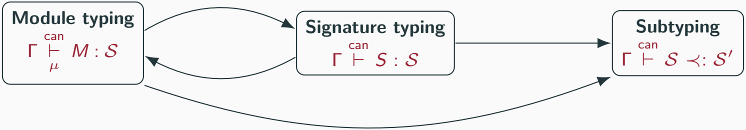
The canonical system



C-Typ-Mod-AppFct

$$\frac{Y \notin \Gamma}{\Gamma \overset{\text{can}}{\underset{\mu}{\vdash}} (Y : S) \to M :}$$

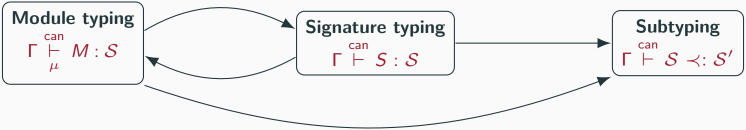
The canonical system



C-Typ-Mod-AppFct

$$\frac{\Gamma \overset{\text{can}}{\vdash} S : \lambda\overline{\alpha}.\mathcal{C} \qquad Y \notin \Gamma}{\Gamma \overset{\text{can}}{\underset{\mu}{\vdash}} (Y : S) \to M :}$$

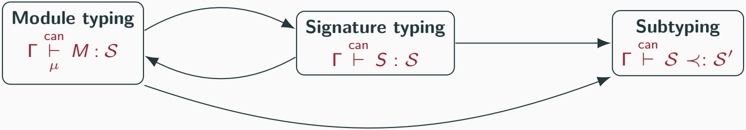
The canonical system



C-Typ-Mod-AppFct

$$\frac{\Gamma \overset{\text{can}}{\vdash} S : \lambda\overline{\alpha}.\mathcal{C} \qquad \Gamma, \overline{\alpha}, (Y : \mathcal{C}) \overset{\text{can}}{\underset{\text{app}}{\vdash}} M : \exists\overline{\beta}.\,(\tau, \mathcal{R}) \qquad Y \notin \Gamma}{\Gamma \overset{\text{can}}{\underset{\mu}{\vdash}} (Y : S) \to M :}$$

The canonical system



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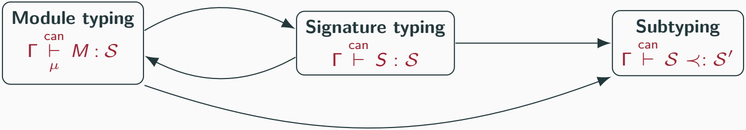
The canonical system



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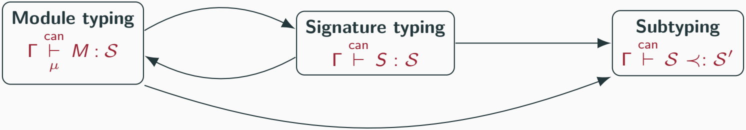
The canonical system



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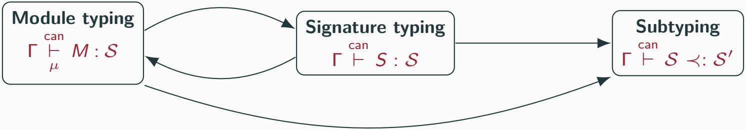

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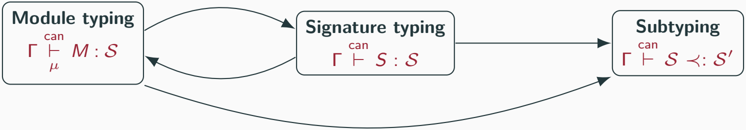
The canonical system



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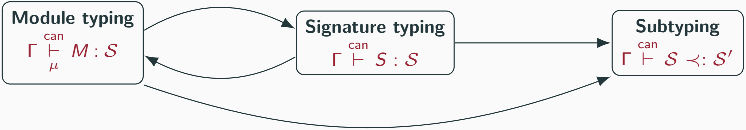
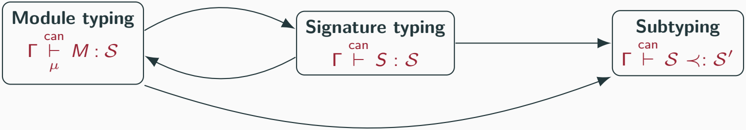
The canonical system



C-Typ-Mod-Proj

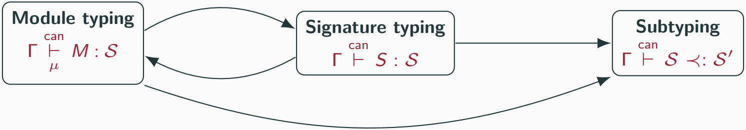
$$\frac{}{\Gamma \overset{\text{can}}{\underset{\mu}{\vdash}} M.X :}$$



C-Typ-Mod-Proj

$$\frac{\Gamma \overset{\text{can}}{\underset{\mu}{\vdash}} M : \exists \overline{\alpha}.\left(\tau, \texttt{sig}\ \overline{\mathcal{D}}\ \texttt{end}\right)}{\Gamma \overset{\text{can}}{\underset{\mu}{\vdash}} M.X :}$$

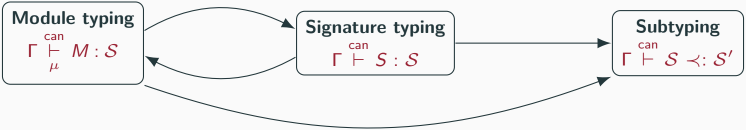
The canonical system



C-TYP-MOD-PROJ

$$\frac{\Gamma \overset{\text{can}}{\underset{\mu}{\vdash}} M : \exists \overline{\alpha}.\ \left(\tau, \texttt{sig}\ \overline{\mathcal{D}}\ \texttt{end}\right) \qquad \text{module}\, X : \mathcal{C} \in \overline{D}}{\Gamma \overset{\text{can}}{\underset{\mu}{\vdash}} M.X :}$$

The canonical system



C-Typ-Mod-Proj

$$\frac{\Gamma \overset{\text{can}}{\underset{\mu}{\vdash}} M : \exists\overline{\alpha}.\left(\tau, \texttt{sig}\ \overline{\mathcal{D}}\ \texttt{end}\right) \qquad \text{module}\, X : \mathcal{C} \in \overline{D}}{\Gamma \overset{\text{can}}{\underset{\mu}{\vdash}} M.X : \exists\overline{\alpha}.\mathcal{C}}$$

A rich history of ML-modules

First formalizations of module systems

A rich history of ML-modules

First formalizations of module systems

- SML modules: Harper et al. [1989]

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Existential types for modules

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Formalization via elaboration

- **Translation of a significant part of SML into F^{ω}: F-ing Rossberg et al. [2014]**
- Core and module languages merged in F^{ω}: 1ML Rossberg [2018]

A user friendly but limited syntax: the source system

Anchoring - a reverse process 1/3

Abstract type fields

```
(* Canonical *)
∃α. module M : sig
  type t = α
  val x : α
  ...
end
```

```
(* Source *)
module M : sig



end
```

Anchoring - a reverse process 1/3

Abstract type fields

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  type t = α
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  ...
end
```

```
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  val x : t
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Anchoring - a reverse process 1/3

Abstract type fields

- Merge quantifier and structural information

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```
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Anchoring - a reverse process 1/3

Abstract type fields

- Merge quantifier and structural information
- The first *usage point* must be suitable to give a path

```
(* Canonical *)
∃α. module M : sig
  type t = α
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  ...
end
```

```
(* Source *)
module M : sig
  type t
  val x : t
  ...
end
```

Anchoring - a reverse process 1/3

Abstract type fields

- Merge quantifier and structural information
- The first *usage point* must be suitable to give a path

```
(* Canonical *)
∃α. module M : sig
  type t = α * bool
  val x : α
  ...
end
```

```
(* Source *)
module M : sig
  type t ✗
  val x : t
  ...
end
```

Anchoring - a reverse process 1/3

Abstract type fields

- Merge quantifier and structural information
- The first *usage point* must be suitable to give a path

> Anchoring map $\theta : \alpha \mapsto P.t$

```
(* Canonical *)
∃α. module M : sig
  type t = α * bool
  val x : α
  ...
end
```

```
(* Source *)
module M : sig
  type t ✗
  val x : t
  ...
end
```

Anchoring - a reverse process 2/3

Module identities

```
(* Canonical *)
∃α_X. module M : sig


    ...
end
```

```
(* Source *)
module M : sig



end
```

Anchoring - a reverse process 2/3

Module identities

```
(* Canonical *)
∃α_X. module M : sig
  module X1 : (α_X, R_1)

    ...
end
```

```
(* Source *)
module M : sig


    ...
end
```

Anchoring - a reverse process 2/3

Module identities

```
(* Canonical *)
∃α_X. module M : sig
  module X1 : (α_X, R_1)
  module X2 : (α_X, R_2)
   ...
end
```

```
(* Source *)
module M : sig


   ...
end
```

Anchoring - a reverse process 2/3

Module identities

- Identity sharing is only expressed as restriction of a common ancestor

```
(* Canonical *)
∃α_X. module M : sig
  module X1 : (α_X, R_1)
  module X2 : (α_X, R_2)
   ...
end
```

$\exists \alpha_X.$ **module** M : **sig** / **module** X1 : $(\alpha_X, \mathcal{R}_1)$ / **module** X2 : $(\alpha_X, \mathcal{R}_2)$

```
(* Source *)
module M : sig
  module X1 : S1

   ...
end
```

Anchoring - a reverse process 2/3

Module identities

- Identity sharing is only expressed as restriction of a common ancestor

```
(* Canonical *)
∃α_X. module M : sig
  module X1 : (α_X, R_1)
  module X2 : (α_X, R_2)
   ...
end
```

```
(* Source *)
module M : sig
  module X1 : S1
  module X2 : (= X1 < S2)
   ...
end
```

Anchoring - a reverse process 2/3

Module identities

- Identity sharing is only expressed as restriction of a common ancestor
- All identities must be ascriptions of the anchoring point

(∗ Canonical ∗)
$\exists \alpha_X$. **module** M : **sig**
  module X1 : $(\alpha_X, \mathcal{R}_1)$
  module X2 : $(\alpha_X, \mathcal{R}_2)$
  ...
end

(∗ Source ∗)
module M : **sig**
  module X1 : S1
  module X2 : $(=$ X1 $<$ S2$)$
  ...
end

Anchoring - a reverse process 2/3

Module identities

- Identity sharing is only expressed as restriction of a common ancestor
- All identities must be ascriptions of the anchoring point

> Anchoring map $\theta : \alpha \mapsto (P, \mathcal{R})$

(* Canonical *)
$\exists \alpha_X$. **module** M : **sig**
  module X1 : $(\alpha_X, \mathcal{R}_1)$
  module X2 : $(\alpha_X, \mathcal{R}_2)$
   ...
end

(* Source *)
module M : **sig**
  module X1 : S1
  module X2 : (= X1 $<$ S2)
   ...
end

Anchoring - a reverse process 3/3

High order abstract types

```
(* Canonical *)
∃γ. module M : sig
end
```

```
(* Source *)
module M : sig
end
```

Anchoring - a reverse process 3/3

High order abstract types

```
(* Canonical *)
∃γ. module M : sig
  module F1 : ∀β, C₁ →



end
```

```
(* Source *)
module M : sig




end
```

Anchoring - a reverse process 3/3

High order abstract types

```
(* Canonical *)
∃γ. module M : sig
  module F1 : ∀β, C_1 →
    sig type t = γ(β) end


end
```

```
(* Source *)
module M : sig





end
```

Anchoring - a reverse process 3/3

High order abstract types

```
(* Canonical *)
∃γ. module M : sig
  module F1 : ∀β, C₁ →
    sig type t = γ(β) end
  module F2 : ∀β, C₂ →

end
```

```
(* Source *)
module M : sig




end
```

High order abstract types

```
(* Canonical *)
∃γ. module M : sig
  module F1 : ∀β,C1 →
    sig type t = γ(β) end
  module F2 : ∀β,C2 →
    sig type t = γ(β) end
end
```

```
(* Source *)
module M : sig




end
```

High order abstract types

- Type operators are only represented as functor applications

```
(* Canonical *)
∃γ. module M : sig
  module F1 : ∀β, C₁ →
    sig type t = γ(β) end
  module F2 : ∀β, C₂ →
    sig type t = γ(β) end
end
```

```
(* Source *)
module M : sig




end
```

Anchoring - a reverse process 3/3

High order abstract types

- Type operators are only represented as functor applications

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(* Canonical *)
∃γ. module M : sig
  module F1 : ∀β, C₁ →
    sig type t = γ(β) end
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    sig type t = γ(β) end
end
```

```
(* Source *)
module M : sig
  module F1 : functor (Y:S1) ->



end
```

Anchoring - a reverse process 3/3

High order abstract types

- Type operators are only represented as functor applications

```
(* Canonical *)
∃γ. module M : sig
  module F1 : ∀β, C₁ →
    sig type t = γ(β) end
  module F2 : ∀β, C₂ →
    sig type t = γ(β) end
end
```

```
(* Source *)
module M : sig
  module F1 : functor (Y:S1) ->
    sig type t end


end
```

Anchoring - a reverse process 3/3

High order abstract types

- Type operators are only represented as functor applications

```
(* Canonical *)
∃γ. module M : sig
  module F1 : ∀β, C1 →
    sig type t = γ(β) end
  module F2 : ∀β, C2 →
    sig type t = γ(β) end
end
```

```
(* Source *)
module M : sig
  module F1 : functor (Y:S1) ->
    sig type t end
  module F2 : functor (Y:S2) ->

end
```

Anchoring - a reverse process 3/3

High order abstract types

- Type operators are only represented as functor applications

```
(* Canonical *)
∃γ. module M : sig
  module F1 : ∀β, C_1 →
    sig type t = γ(β) end
  module F2 : ∀β, C_2 →
    sig type t = γ(β) end
end
```

```
(* Source *)
module M : sig
  module F1 : functor (Y:S1) ->
    sig type t end
  module F2 : functor (Y:S2) ->
    sig type t = F1(Y).t end
end
```

Anchoring - a reverse process 3/3

High order abstract types

- Type operators are only represented as functor applications
- All usages must be obtainable by some application of the *anchoring point*

```
(* Canonical *)
∃γ. module M : sig
  module F1 : ∀β, C_1 →
    sig type t = γ(β) end
  module F2 : ∀β, C_2 →
    sig type t = γ(β) end
end
```

```
(* Source *)
module M : sig
  module F1 : functor (Y:S1) ->
    sig type t end
  module F2 : functor (Y:S2) ->
    sig type t = F1(Y).t end
end
```

Anchoring - a reverse process 3/3

High order abstract types

- Type operators are only represented as functor applications
- All usages must be obtainable by some application of the *anchoring point*

> Anchoring map $\theta : \alpha \mapsto \lambda\overline{\alpha}.\lambda\overline{X}.(P.t, \mathcal{R}, \overline{\tau})$

```
(* Canonical *)
∃γ. module M : sig
  module F1 : ∀β, C_1 →
    sig type t = γ(β) end
  module F2 : ∀β, C_2 →
    sig type t = γ(β) end
end
```

```
(* Source *)
module M : sig
  module F1 : functor (Y:S1) ->
    sig type t end
  module F2 : functor (Y:S2) ->
    sig type t = F1(Y).t end
end
```

Challenges for a source system

Challenges for a source system

Signature avoidance

Challenges for a source system

Signature avoidance

- abstract type fields (possibly high-order)

Challenges for a source system

Signature avoidance

- abstract type fields (possibly high-order)
- module identities

Challenges for a source system

Signature avoidance

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Implicit quantifiers

Challenges for a source system

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- Strengthening (deep rewrites)

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Implicit quantifiers

- Strengthening (deep rewrites)

Type and signatures equivalence

Challenges for a source system

Signature avoidance

- abstract type fields (possibly high-order)
- module identities

Implicit quantifiers

- Strengthening (deep rewrites)

Type and signatures equivalence

- Types have several available aliases

Challenges for a source system

Signature avoidance

- abstract type fields (possibly high-order)
- module identities

Implicit quantifiers

- Strengthening (deep rewrites)

Type and signatures equivalence

- Types have several available aliases
- Prevent in-lining of module types

Elaboration into F^{ω}: guarantees for the canonical system

Example of encoding into $\mathbf{F}^\omega$

Example of encoding into F^ω

Source code

```
module M = struct
  module X1 : S = struct
    type t = int
    let x = 42
    type u = int * t
  end
  module X2 = struct
    type t = X1.t * bool
    type u = X1.t * int
  end
end
```

Example of encoding into F^ω

Source code

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module M = struct
  module X1 : S = struct
    type t = int
    let x = 42
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  end
  module X2 = struct
    type t = X1.t * bool
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  end
end
```

Encoded signature

Example of encoding into F^ω

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Encoded signature

$$\Pi = \exists \alpha$$

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Encoded signature

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  end
  module X2 = struct
    type t = X1.t * bool
    type u = X1.t * int
  end
end
```

Encoded signature

$$\Pi = \exists \alpha \left\{ \quad \ell_{X1} : \left\{ \quad \right. \right.$$

Example of encoding into F^ω

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```
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  module X1 : S = struct
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  end
  module X2 = struct
    type t = X1.t * bool
    type u = X1.t * int
  end
end
```

Encoded signature

$$\Pi = \exists\alpha \left\{ \quad \ell_{X1} \colon \left\{ \begin{array}{l} \ell_t : \langle\!\langle \alpha \rangle\!\rangle \\ \\ \end{array} \right. \\ \\ \right.$$

Example of encoding into F^ω

Source code

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  module X1 : S = struct
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    type u = int * t
  end
  module X2 = struct
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  end
end
```

Encoded signature

$$\Pi = \exists \alpha \left\{ \ell_{X1} : \left\{ \ell_t : \forall \beta . \beta\, \alpha \to \beta\, \alpha \right. \right.$$

Example of encoding into F^{ω}

Source code

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module M = struct
  module X1 : S = struct
    type t = int
    let x = 42
    type u = int * t
  end
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    type t = X1.t * bool
    type u = X1.t * int
  end
end
```

Encoded signature

$$\Pi = \exists \alpha \left\{ \quad \ell_{X1} \colon \begin{cases} \ell_t : \langle\!\langle \alpha \rangle\!\rangle \\ \ell_x : \alpha \\ \end{cases} \right.$$

Example of encoding into F^ω

Source code

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    let x = 42
    type u = int * t
  end
  module X2 = struct
    type t = X1.t * bool
    type u = X1.t * int
  end
end
```

Encoded signature

$$\Pi = \exists \alpha \left\{ \ell_{X1} : \begin{cases} \ell_t : \langle\!\langle \alpha \rangle\!\rangle \\ \ell_x : \alpha \\ \ell_u : \langle\!\langle \mathtt{int} \times \alpha \rangle\!\rangle \end{cases} \right.$$

Example of encoding into F^ω

Source code

```
module M = struct
  module X1 : S = struct
    type t = int
    let x = 42
    type u = int * t
  end
  module X2 = struct
    type t = X1.t * bool
    type u = X1.t * int
  end
end
```

Encoded signature

$$\Pi = \exists\alpha \left\{ \quad \ell_{X1} \colon \left\{ \begin{array}{l} \ell_t : \langle\!\langle \alpha \rangle\!\rangle \\ \ell_x : \alpha \\ \ell_u : \langle\!\langle \mathtt{int} \times \alpha \rangle\!\rangle \end{array} \right\} \right.$$

Example of encoding into F^{ω}

Source code

```
module M = struct
  module X1 : S = struct
    type t = int
    let x = 42
    type u = int * t
  end
  module X2 = struct
    type t = X1.t * bool
    type u = X1.t * int
  end
end
```

Encoded signature

$$\Pi = \exists\alpha \left\{ \begin{array}{l} \ell_{X1} : \left\{ \begin{array}{l} \ell_t : \langle\!\langle \alpha \rangle\!\rangle \\ \ell_x : \alpha \\ \ell_u : \langle\!\langle \mathtt{int} \times \alpha \rangle\!\rangle \end{array} \right\} \\ \\ \ell_{X2} : \left\{ \begin{array}{l} \\ \\ \end{array} \right. \end{array} \right.$$

Example of encoding into F^ω

Source code

```
module M = struct
  module X1 : S = struct
    type t = int
    let x = 42
    type u = int * t
  end
  module X2 = struct
    type t = X1.t * bool
    type u = X1.t * int
  end
end
```

Encoded signature

$$\Pi = \exists \alpha \left\{ \begin{array}{l} \ell_{X1} : \left\{ \begin{array}{l} \ell_t : \langle\!\langle \alpha \rangle\!\rangle \\ \ell_x : \alpha \\ \ell_u : \langle\!\langle \mathtt{int} \times \alpha \rangle\!\rangle \end{array} \right\} \\ \ell_{X2} : \left\{ \begin{array}{l} \ell_t : \langle\!\langle \alpha \times \mathtt{bool} \rangle\!\rangle \\ \end{array} \right. \end{array} \right.$$

Example of encoding into F^ω

Source code

```
module M = struct
  module X1 : S = struct
    type t = int
    let x = 42
    type u = int * t
  end
  module X2 = struct
    type t = X1.t * bool
    type u = X1.t * int
  end
end
```

Encoded signature

$$\Pi = \exists\alpha \left\{ \begin{array}{l} \ell_{X1}\colon \left\{ \begin{array}{l} \ell_t : \langle\!\langle \alpha \rangle\!\rangle \\ \ell_x : \alpha \\ \ell_u : \langle\!\langle \mathtt{int} \times \alpha \rangle\!\rangle \end{array} \right\} \\ \ell_{X2}\colon \left\{ \begin{array}{l} \ell_t : \langle\!\langle \alpha \times \mathtt{bool} \rangle\!\rangle \\ \ell_u : \langle\!\langle \alpha \times \mathtt{int} \rangle\!\rangle \end{array} \right. \end{array} \right.$$

Example of encoding into F^ω

Source code

```
module M = struct
  module X1 : S = struct
    type t = int
    let x = 42
    type u = int * t
  end
  module X2 = struct
    type t = X1.t * bool
    type u = X1.t * int
  end
end
```

Encoded signature

$$\Pi = \exists \alpha \left\{ \begin{array}{l} \ell_{X1}\colon \left\{ \begin{array}{l} \ell_t : \langle\!\langle \alpha \rangle\!\rangle \\ \ell_x : \alpha \\ \ell_u : \langle\!\langle \mathtt{int} \times \alpha \rangle\!\rangle \end{array} \right\} \\ \ell_{X2}\colon \left\{ \begin{array}{l} \ell_t : \langle\!\langle \alpha \times \mathtt{bool} \rangle\!\rangle \\ \ell_u : \langle\!\langle \alpha \times \mathtt{int} \rangle\!\rangle \end{array} \right\} \end{array} \right.$$

Example of encoding into F^ω

Source code

```
module M = struct
  module X1 : S = struct
    type t = int
    let x = 42
    type u = int * t
  end
  module X2 = struct
    type t = X1.t * bool
    type u = X1.t * int
  end
end
```

Encoded signature

$$\Pi = \exists \alpha \left\{ \begin{array}{l} \text{module } X_1 : \left\{ \begin{array}{l} \text{type } t = \alpha \\ \text{val } x : \alpha \\ \text{type } u = \texttt{int} \times \alpha \end{array} \right\} \\ \text{module } X_2 : \left\{ \begin{array}{l} \text{type } t = \alpha \times \texttt{bool} \\ \text{type } u = \alpha \times \texttt{int} \end{array} \right\} \end{array} \right\}$$

Example of encoding into F^ω

Source code

```
module M = struct
  module X1 : S = struct
    type t = int
    let x = 42
    type u = int * t
  end
  module X2 = struct
    type t = X1.t * bool
    type u = X1.t * int
  end
end
```

Encoded module

Encoded signature

$$\Pi = \exists\alpha \left\{ \begin{array}{l} \text{module } X_1 : \left\{ \begin{array}{l} \text{type } t = \alpha \\ \text{val } x : \alpha \\ \text{type } u = \texttt{int} \times \alpha \end{array} \right\} \\ \text{module } X_2 : \left\{ \begin{array}{l} \text{type } t = \alpha \times \texttt{bool} \\ \text{type } u = \alpha \times \texttt{int} \end{array} \right\} \end{array} \right\}$$

Example of encoding into F^ω

Source code

```
module M = struct
  module X1 : S = struct
    type t = int
    let x = 42
    type u = int * t
  end
  module X2 = struct
    type t = X1.t * bool
    type u = X1.t * int
  end
end
```

Encoded signature

$$\Pi = \exists\alpha \left\{ \begin{array}{l} \text{module } X_1 : \left\{ \begin{array}{l} \text{type } t = \alpha \\ \text{val } x : \alpha \\ \text{type } u = \texttt{int} \times \alpha \end{array} \right\} \\ \text{module } X_2 : \left\{ \begin{array}{l} \text{type } t = \alpha \times \texttt{bool} \\ \text{type } u = \alpha \times \texttt{int} \end{array} \right\} \end{array} \right\}$$

Encoded module

$$E =$$

Example of encoding into F$^\omega$

Source code

```
module M = struct
  module X1 : S = struct
    type t = int
    let x = 42
    type u = int * t
  end
  module X2 = struct
    type t = X1.t * bool
    type u = X1.t * int
  end
end
```

Encoded signature

$$\Pi = \exists\alpha \left\{ \begin{array}{l} \text{module } X_1 : \left\{ \begin{array}{l} \text{type } t = \alpha \\ \text{val } x : \alpha \\ \text{type } u = \texttt{int} \times \alpha \end{array} \right\} \\ \text{module } X_2 : \left\{ \begin{array}{l} \text{type } t = \alpha \times \texttt{bool} \\ \text{type } u = \alpha \times \texttt{int} \end{array} \right\} \end{array} \right\}$$

Encoded module

$$E = \{\ell_{X1} = \{\ell_t = \langle\langle \texttt{int} \rangle\rangle, \ell_x = 42, \ell_u = \langle\langle \texttt{int} \times \texttt{int} \rangle\rangle\}\}$$

Example of encoding into F^ω

Source code

```
module M = struct
  module X1 : S = struct
    type t = int
    let x = 42
    type u = int * t
  end
  module X2 = struct
    type t = X1.t * bool
    type u = X1.t * int
  end
end
```

Encoded signature

$$\Pi = \exists \alpha \left\{ \begin{array}{l} \text{module } X_1 : \left\{ \begin{array}{l} \text{type } t = \alpha \\ \text{val } x : \alpha \\ \text{type } u = \texttt{int} \times \alpha \end{array} \right\} \\ \text{module } X_2 : \left\{ \begin{array}{l} \text{type } t = \alpha \times \texttt{bool} \\ \text{type } u = \alpha \times \texttt{int} \end{array} \right\} \end{array} \right\}$$

Encoded module

$$E = \text{pack}\langle \texttt{int}, \{\ell_{X1} = \{\ell_t = \langle\langle \texttt{int} \rangle\rangle, \ell_x = 42, \ell_u = \langle\langle \texttt{int} \times \texttt{int} \rangle\rangle\}\}\rangle$$

Example of encoding into F^ω

Source code

```
module M = struct
  module X1 : S = struct
    type t = int
    let x = 42
    type u = int * t
  end
  module X2 = struct
    type t = X1.t * bool
    type u = X1.t * int
  end
end
```

Encoded signature

$$\Pi = \exists\alpha \left\{ \begin{array}{l} \text{module } X_1 : \left\{ \begin{array}{l} \text{type } t = \alpha \\ \text{val } x : \alpha \\ \text{type } u = \texttt{int} \times \alpha \end{array} \right\} \\ \text{module } X_2 : \left\{ \begin{array}{l} \text{type } t = \alpha \times \texttt{bool} \\ \text{type } u = \alpha \times \texttt{int} \end{array} \right\} \end{array} \right\}$$

Encoded module

$$E = \text{pack}\langle \texttt{int}, \{\ell_{X1} = \{\ell_t = \langle\!\langle \texttt{int} \rangle\!\rangle, \ell_x = 42, \ell_u = \langle\!\langle \texttt{int} \times \texttt{int} \rangle\!\rangle\}\}\rangle$$
$$\{\ell_{X2} = \{\ell_t = \langle\!\langle \alpha \times \texttt{bool} \rangle\!\rangle, \ell_u = \langle\!\langle \alpha \times \texttt{int} \rangle\!\rangle\}\}$$

Example of encoding into F^ω

Source code

```
module M = struct
  module X1 : S = struct
    type t = int
    let x = 42
    type u = int * t
  end
  module X2 = struct
    type t = X1.t * bool
    type u = X1.t * int
  end
end
```

Encoded signature

$$\Pi = \exists\alpha \left\{ \begin{array}{l} \text{module } X_1 : \left\{ \begin{array}{l} \text{type } t = \alpha \\ \text{val } x : \alpha \\ \text{type } u = \texttt{int} \times \alpha \end{array} \right\} \\ \text{module } X_2 : \left\{ \begin{array}{l} \text{type } t = \alpha \times \texttt{bool} \\ \text{type } u = \alpha \times \texttt{int} \end{array} \right\} \end{array} \right\}$$

Encoded module

$$\begin{array}{l} E = \text{unpack}\langle \alpha, y_1 \rangle = \text{pack}\langle \texttt{int}, \{\ell_{X1} = \{\ell_t = \langle\!\langle \texttt{int} \rangle\!\rangle, \ell_x = 42, \ell_u = \langle\!\langle \texttt{int} \times \texttt{int} \rangle\!\rangle\}\}\rangle \\ \quad\quad \text{in} \quad\quad\quad\quad \{\ell_{X2} = \{\ell_t = \langle\!\langle \alpha \times \texttt{bool} \rangle\!\rangle, \ell_u = \langle\!\langle \alpha \times \texttt{int} \rangle\!\rangle\}\} \end{array}$$

Example of encoding into F^ω

Source code

```
module M = struct
  module X1 : S = struct
    type t = int
    let x = 42
    type u = int * t
  end
  module X2 = struct
    type t = X1.t * bool
    type u = X1.t * int
  end
end
```

Encoded signature

$$\Pi = \exists\alpha \left\{ \begin{array}{l} \text{module } X_1 : \left\{ \begin{array}{l} \text{type } t = \alpha \\ \text{val } x : \alpha \\ \text{type } u = \texttt{int} \times \alpha \end{array} \right\} \\ \text{module } X_2 : \left\{ \begin{array}{l} \text{type } t = \alpha \times \texttt{bool} \\ \text{type } u = \alpha \times \texttt{int} \end{array} \right\} \end{array} \right\}$$

Encoded module

$$\begin{array}{l} E = \text{unpack}\langle \alpha, y_1 \rangle = \text{pack}\langle \texttt{int}, \{\ell_{X1} = \{\ell_t = \langle\!\langle \texttt{int} \rangle\!\rangle, \ell_x = 42, \ell_u = \langle\!\langle \texttt{int} \times \texttt{int} \rangle\!\rangle\}\}\rangle \\ \quad \text{in unpack}\langle \varnothing, y_2 \rangle = \{\ell_{X2} = \{\ell_t = \langle\!\langle \alpha \times \texttt{bool} \rangle\!\rangle, \ell_u = \langle\!\langle \alpha \times \texttt{int} \rangle\!\rangle\}\} \\ \quad\quad \text{in} \end{array}$$

Example of encoding into F^{ω}

Source code

```
module M = struct
  module X1 : S = struct
    type t = int
    let x = 42
    type u = int * t
  end
  module X2 = struct
    type t = X1.t * bool
    type u = X1.t * int
  end
end
```

Encoded signature

$$\Pi = \exists\alpha \left\{ \begin{array}{l} \text{module } X_1 : \left\{ \begin{array}{l} \text{type } t = \alpha \\ \text{val } x : \alpha \\ \text{type } u = \texttt{int} \times \alpha \end{array} \right\} \\ \text{module } X_2 : \left\{ \begin{array}{l} \text{type } t = \alpha \times \texttt{bool} \\ \text{type } u = \alpha \times \texttt{int} \end{array} \right\} \end{array} \right\}$$

Encoded module

$$\begin{array}{l} E = \text{unpack}\langle \alpha, y_1 \rangle = \text{pack}\langle \texttt{int}, \{\ell_{X1} = \{\ell_t = \langle\!\langle \texttt{int} \rangle\!\rangle, \ell_x = 42, \ell_u = \langle\!\langle \texttt{int} \times \texttt{int} \rangle\!\rangle\}\}\rangle \\ \quad \text{in unpack}\langle \varnothing, y_2 \rangle = \{\ell_{X2} = \{\ell_t = \langle\!\langle \alpha \times \texttt{bool} \rangle\!\rangle, \ell_u = \langle\!\langle \alpha \times \texttt{int} \rangle\!\rangle\}\} \\ \quad\quad \text{in pack}\langle \alpha, \{\ell_{X1} = (y_1.\ell_{X1}), \ell_{X2} = (y_2.\ell_{X2})\}\rangle \end{array}$$

The issue of *skolemisation*

Source code

```
module M = struct
  module X1 : S = struct
    type t = int
    let x = 42
    type u = int * t
  end
  module X2 = struct
    type t = X1.t * bool
    type u = X1.t * int
  end
end
```

Encoded signature

$$\Pi = \exists \alpha . \mathcal{R}$$

Encoded module

$$E = \mathsf{pack}\langle \alpha, \ldots \rangle$$

The issue of *skolemisation*

Source code

```
module M (Y:S) = struct
  module X1 : S = struct
    type t = int
    let x = 42
    type u = int * t
  end
  module X2 = struct
    type t = X1.t * bool
    type u = X1.t * int
  end
end
```

Encoded signature

$$\Pi = \exists \alpha . \mathcal{R}$$

Encoded module

$$E = \mathsf{pack}\langle \alpha, \dots \rangle$$

The issue of *skolemisation*

Source code

```
module M (Y:S) = struct
  module X1 : S = struct
    type t = int
    let x = 42
    type u = int * t
  end
  module X2 = struct
    type t = X1.t * bool
    type u = X1.t * int
  end
end
```

Encoded signature

$$\Pi = \forall \beta . \mathcal{C} \rightarrow \exists \alpha . \mathcal{R}$$

Encoded module

$$E = \mathsf{pack}\langle \alpha, \ldots \rangle$$

The issue of *skolemisation*

Source code

```
module M (Y:S) = struct
  module X1 : S = struct
    type t = int
    let x = 42
    type u = int * t
  end
  module X2 = struct
    type t = X1.t * bool
    type u = X1.t * int
  end
end
```

Encoded signature

$$\Pi = \exists \alpha'.\forall \beta.\mathcal{C} \rightarrow \mathcal{R}\Big[\alpha \mapsto \alpha'(\beta)\Big]$$

Encoded module

$$E = \mathsf{pack}\langle \alpha, \ldots \rangle$$

The issue of *skolemisation*

Source code

```
module M (Y:S) = struct
  module X1 : S = struct
    type t = int
    let x = 42
    type u = int * t
  end
  module X2 = struct
    type t = X1.t * bool
    type u = X1.t * int
  end
end
```

Encoded signature

$$\Pi = \exists\alpha'.\forall\beta.\mathcal{C} \to \mathcal{R}\Big[\alpha \mapsto \alpha'(\beta)\Big]$$

Encoded module

$$E = \Lambda\beta.\lambda(Y : \mathcal{C}).\mathsf{pack}\langle\alpha, \dots\rangle$$

The issue of *skolemisation*

Source code

```
module M (Y:S) = struct
  module X1 : S = struct
    type t = int
    let x = 42
    type u = int * t
  end
  module X2 = struct
    type t = X1.t * bool
    type u = X1.t * int
  end
end
```

Encoded signature

$$\Pi = \exists \alpha'.\forall \beta.\mathcal{C} \to \mathcal{R}\Big[\alpha \mapsto \alpha'(\beta)\Big]$$

Encoded module

$$E = \Lambda \beta.\lambda(Y : \mathcal{C}).\mathsf{pack}\langle \alpha, \dots \rangle$$

The issue of *skolemisation*

Source code

```
module M (Y:S) = struct
  module X1 : S = struct
    type t = int
    let x = 42
    type u = int * t
  end
  module X2 = struct
    type t = X1.t * bool
    type u = X1.t * int
  end
end
```

Encoded signature

$$\Pi = \forall \beta.\mathcal{C} \rightarrow \exists (\alpha = \tau).\mathcal{R}$$

Encoded module

$$E = \Lambda \beta.\lambda (Y : \mathcal{C}).\mathsf{pack}\langle \alpha, \ldots \rangle$$

The issue of ***skolemisation***

Source code

```
module M (Y:S) = struct
  module X1 : S = struct
    type t = int
    let x = 42
    type u = int * t
  end
  module X2 = struct
    type t = X1.t * bool
    type u = X1.t * int
  end
end
```

Encoded signature

$$\Pi = \exists(\alpha' = \lambda\beta.\tau).\forall\beta.\mathcal{C} \to \mathcal{R}\left[\alpha \mapsto \alpha'(\beta)\right]$$

Encoded module

$$E = \Lambda\beta.\lambda(Y : \mathcal{C}).\mathsf{pack}\langle\alpha, \dots\rangle$$

The issue of *skolemisation*

Source code

```
module M (Y:S) = struct
  module X1 : S = struct
    type t = int
    let x = 42
    type u = int * t
  end
  module X2 = struct
    type t = X1.t * bool
    type u = X1.t * int
  end
end
```

Encoded signature

$$\Pi = \exists(\alpha' = \lambda\beta.\tau).\forall\beta.\mathcal{C} \to \mathcal{R}\Big[\alpha \mapsto \alpha'(\beta)\Big]$$

Encoded module

$$E = \mathsf{pack}\langle\alpha, \dots\rangle$$

The issue of *skolemisation*

Source code

```
module M (Y:S) = struct
  module X1 : S = struct
    type t = int
    let x = 42
    type u = int * t
  end
  module X2 = struct
    type t = X1.t * bool
    type u = X1.t * int
  end
end
```

Encoded signature

$$\Pi = \exists(\alpha' = \lambda\beta.\tau).\forall\beta.\mathcal{C} \to \mathcal{R}\Big[\alpha \mapsto \alpha'(\beta)\Big]$$

Encoded module

$$E = \ldots \text{ as } \exists(\alpha = \tau).\mathcal{R}$$

The issue of *skolemisation*

Source code

```
module M (Y:S) = struct
  module X1 : S = struct
    type t = int
    let x = 42
    type u = int * t
  end
  module X2 = struct
    type t = X1.t * bool
    type u = X1.t * int
  end
end
```

Encoded signature

$$\Pi = \exists(\alpha' = \lambda\beta.\tau).\forall\beta.\mathcal{C} \to \mathcal{R}\Big[\alpha \mapsto \alpha'(\beta)\Big]$$

Encoded module

$$E = \text{skolem}_{\forall}(\Lambda\beta.\text{skolem}_{\to}(\lambda(Y : \mathcal{C}).(\ldots \text{ as } \exists(\alpha = \tau).\mathcal{R})))$$

F^{ω} with skolemisation

New constructs

F^ω with skolemisation

New constructs

$$\begin{aligned} \tau ::= &\ \dots \\ &\mid \exists\alpha.\tau \end{aligned}$$

F^{ω} with skolemisation

New constructs

$$\begin{aligned}\tau ::= &\ \ldots \\ &\mid \exists \alpha . \tau \\ &\mid \exists \left(\alpha = \tau\right) . \tau\end{aligned}$$

F^ω with skolemisation

New constructs

$$
\begin{aligned}
\tau ::= \;& \ldots \\
\mid\;& \exists \alpha . \tau \\
\mid\;& \exists (\alpha = \tau) . \tau \\
e ::= \;& \ldots \\
\mid\;& \mathsf{pack}\, e \\
\mid\;& \mathsf{unpack} \langle \alpha, x \rangle = e \ \mathsf{in}\ e
\end{aligned}
$$

F^ω with skolemisation

New constructs

$$
\begin{aligned}
\tau ::=\ & \dots \\
& \mid \exists \alpha . \tau \\
& \mid \exists \left(\alpha = \tau \right) . \tau \\
e ::=\ & \dots \\
& \mid \mathsf{pack}\, e \\
& \mid \mathsf{unpack} \langle \alpha, x \rangle = e \text{ in } e \\
& \mid e \text{ as } \exists (\alpha = \tau) . \tau
\end{aligned}
$$

F^ω with skolemisation

New constructs

$$
\begin{aligned}
\tau ::= &\ \ldots \\
&\mid \exists \alpha.\tau \\
&\mid \exists\left(\alpha = \tau\right).\tau \\
e ::= &\ \ldots \\
&\mid \mathsf{pack}\ e \\
&\mid \mathsf{unpack}\langle \alpha, x\rangle = e\ \mathsf{in}\ e \\
&\mid e\ \mathsf{as}\ \exists(\alpha = \tau).\tau \\
&\mid \mathsf{hide}\langle \alpha, x\rangle = e\ \mathsf{in}\ e
\end{aligned}
$$

F^ω with skolemisation

New constructs

$$
\begin{aligned}
\tau ::=\;& \ldots \\
& \mid \exists \alpha.\tau \\
& \mid \exists\,(\alpha = \tau)\,.\tau \\
e ::=\;& \ldots \\
& \mid \mathsf{pack}\, e \\
& \mid \mathsf{unpack}\langle \alpha, x\rangle = e \;\mathsf{in}\; e \\
& \mid e \;\mathsf{as}\; \exists(\alpha = \tau).\tau \\
& \mid \mathsf{hide}\langle \alpha, x\rangle = e \;\mathsf{in}\; e \\
& \mid \mathsf{show}\, e
\end{aligned}
$$

$\mathbf{F}^{\omega}$ with skolemisation

New constructs

$$
\begin{aligned}
\tau ::= \; & \ldots \\
& \mid \exists \alpha . \tau \\
& \mid \exists \left(\alpha = \tau \right) . \tau \\
e ::= \; & \ldots \\
& \mid \mathsf{pack}\, e \\
& \mid \mathsf{unpack} \langle \alpha, x \rangle = e \;\mathsf{in}\; e \\
& \mid e \;\mathsf{as}\; \exists (\alpha = \tau) . \tau \\
& \mid \mathsf{hide} \langle \alpha, x \rangle = e \;\mathsf{in}\; e \\
& \mid \mathsf{show}\, e
\end{aligned}
$$

Skolem operators

F^ω with skolemisation

New constructs

$$
\begin{aligned}
\tau ::= &\ \dots \\
&\mid \exists\alpha.\tau \\
&\mid \exists\,(\alpha = \tau)\,.\tau \\
e ::= &\ \dots \\
&\mid \mathsf{pack}\, e \\
&\mid \mathsf{unpack}\langle\alpha, x\rangle = e \;\mathsf{in}\; e \\
&\mid e \;\mathsf{as}\; \exists(\alpha = \tau).\tau \\
&\mid \mathsf{hide}\langle\alpha, x\rangle = e \;\mathsf{in}\; e \\
&\mid \mathsf{show}\, e
\end{aligned}
$$

Skolem operators

- $\mathsf{skolem}_{\rightarrow}$:

$$
\begin{aligned}
\sigma &\quad\rightarrow\quad \exists(\alpha = \tau).\rho \\
\exists(\alpha = \tau).\sigma &\quad\rightarrow\quad \rho
\end{aligned}
$$

F^ω with skolemisation

New constructs

$$
\begin{aligned}
\tau ::=\ & \ldots \\
& \mid \exists\alpha.\tau \\
& \mid \exists\,(\alpha = \tau)\,.\tau \\
e ::=\ & \ldots \\
& \mid \mathsf{pack}\, e \\
& \mid \mathsf{unpack}\langle\alpha, x\rangle = e \;\mathsf{in}\; e \\
& \mid e \;\mathsf{as}\; \exists(\alpha = \tau).\tau \\
& \mid \mathsf{hide}\langle\alpha, x\rangle = e \;\mathsf{in}\; e \\
& \mid \mathsf{show}\, e
\end{aligned}
$$

Skolem operators

- $\mathsf{skolem}_{\rightarrow}$:

$$
\begin{aligned}
\sigma &\quad\rightarrow\quad \exists(\alpha = \tau).\rho \\
\exists(\alpha = \tau).\sigma &\quad\rightarrow\quad \rho
\end{aligned}
$$

- $\mathsf{skolem}_{\forall}$

$$
\begin{aligned}
\forall\beta.&\quad \exists(\alpha = \tau).\sigma \\
\exists(\alpha' = \lambda\beta.\tau).\quad \forall\beta.&\quad \sigma[\alpha \mapsto \alpha'(\beta)]
\end{aligned}
$$

F^ω with skolemisation

New constructs

$$
\begin{aligned}
\tau ::= \ & \dots \\
& \mid \exists \alpha . \tau \\
& \mid \exists \, (\alpha = \tau) \, . \tau \\
e ::= \ & \dots \\
& \mid \text{pack}\, e \\
& \mid \text{unpack}\langle \alpha, x \rangle = e \text{ in } e \\
& \mid e \text{ as } \exists (\alpha = \tau) . \tau \\
& \mid \text{hide}\langle \alpha, x \rangle = e \text{ in } e \\
& \mid \text{show}\, e
\end{aligned}
$$

Skolem operators

- $\text{skolem}_{\to}$:

$$
\begin{aligned}
\sigma \quad & \to \quad \exists (\alpha = \tau) . \rho \\
\exists (\alpha = \tau) . \sigma \quad & \to \quad \rho
\end{aligned}
$$

- $\text{skolem}_{\forall}$

$$
\begin{aligned}
& \forall \beta . \quad \exists (\alpha = \tau) . \sigma \\
\exists (\alpha' = \lambda \beta . \tau) . \quad & \forall \beta . \quad \sigma[\alpha \mapsto \alpha'(\beta)]
\end{aligned}
$$

A restricted encoding

$\to$ without show, only with skolem operators

F^ω with skolemisation

New constructs

$$
\begin{aligned}
\tau ::= \; & \ldots \\
& \mid \exists \alpha . \tau \\
& \mid \exists\, (\alpha = \tau)\, . \tau \\
e ::= \; & \ldots \\
& \mid \mathsf{pack}\ e \\
& \mid \mathsf{unpack}\langle \alpha, x \rangle = e \ \mathsf{in}\ e \\
& \mid e \ \mathsf{as}\ \exists (\alpha = \tau) . \tau \\
& \mid \mathsf{hide}\langle \alpha, x \rangle = e \ \mathsf{in}\ e \\
& \mid \mathsf{show}\ e
\end{aligned}
$$

Skolem operators

- $\mathsf{skolem}_{\rightarrow}$:

$$
\begin{aligned}
\sigma \quad & \rightarrow \quad \exists (\alpha = \tau) . \rho \\
\exists (\alpha = \tau) . \sigma \quad & \rightarrow \quad \rho
\end{aligned}
$$

- $\mathsf{skolem}_{\forall}$

$$
\begin{aligned}
\forall \beta . \quad & \exists (\alpha = \tau) . \sigma \\
\exists (\alpha' = \lambda \beta . \tau) . \quad \forall \beta . \quad & \sigma [\alpha \mapsto \alpha'(\beta)]
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A restricted encoding

$\rightarrow$ without show, only with skolem operators

$$
\Gamma \overset{\mathsf{can}}{\underset{\mathsf{app}}{\vdash}} M : \exists \overline{\alpha} . \mathcal{C} \iff \exists \overline{\tau}, e .\ \Gamma \overset{\mathsf{elab}}{\vdash} M : \exists (\overline{\alpha} = \overline{\tau}) . \mathcal{C} \rightsquigarrow e
$$

F^ω with skolemisation

New constructs

$$\begin{aligned}\tau ::= &\ \dots\\ &\mid \exists\alpha.\tau\\ &\mid \exists\,(\alpha = \tau)\,.\tau\\ e ::= &\ \dots\\ &\mid \mathsf{pack}\ e\\ &\mid \mathsf{unpack}\langle\alpha, x\rangle = e\ \mathsf{in}\ e\\ &\mid e\ \mathsf{as}\ \exists(\alpha = \tau).\tau\\ &\mid \mathsf{hide}\langle\alpha, x\rangle = e\ \mathsf{in}\ e\\ &\mid \mathsf{show}\ e\end{aligned}$$

Skolem operators

- $\mathsf{skolem}_{\rightarrow}$:

$$\begin{aligned}\sigma &\quad\rightarrow\quad \exists(\alpha = \tau).\rho\\ \exists(\alpha = \tau).\sigma &\quad\rightarrow\quad \rho\end{aligned}$$

- $\mathsf{skolem}_{\forall}$

$$\begin{aligned}&\forall\beta.\quad \exists(\alpha = \tau).\sigma\\ \exists(\alpha' = \lambda\beta.\tau).\quad &\forall\beta.\quad \sigma[\alpha \mapsto \alpha'(\beta)]\end{aligned}$$

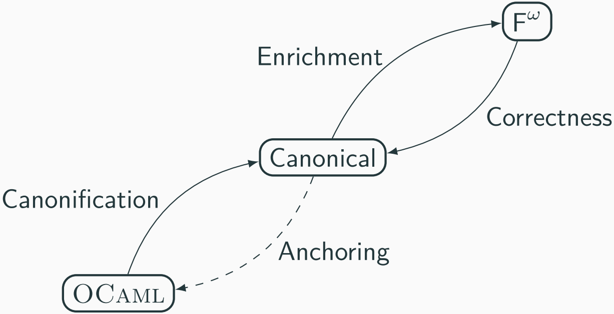
A restricted encoding

$\rightarrow$ without show, only with skolem operators

$$\Gamma \overset{\mathsf{can}}{\underset{\mathsf{app}}{\vdash}} M : \exists\overline{\alpha}.\mathcal{C} \iff \exists\overline{\tau}, e.\ \Gamma \overset{\mathsf{elab}}{\vdash} M : \exists(\overline{\alpha} = \overline{\tau}).\,\mathcal{C} \rightsquigarrow e$$

$$\Gamma \overset{\mathsf{can}}{\underset{\mathsf{gen}}{\vdash}} M : \exists\overline{\alpha}.\mathcal{C} \iff \exists e.\ \Gamma \overset{\mathsf{elab}}{\vdash} M : \exists\overline{\alpha}.\,\mathcal{C} \rightsquigarrow e$$

The big picture



Conclusion and future work

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Takeway

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1. An presentation *spectrum* for OCaml modules, from the current path-based representation to the formal F^{ω} encoding, with the canonical system as a middle-point

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4. Formal guarantees through an improved elaboration into F^ω

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