Partial Graph Reduction:
A New Optimization Technique
for Higher-Order Programs

Lionel Parreaux

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INRIA Paris

Inlining in Optimizing Compilers

Basics of inlining

Consider this program:

```
let f x = x + 7
in f 3 * f 4
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```
(3 + 7) * (4 + 7)
```

Exposing constant folding optimization; resulting in:

```
110
```

Problem

Question: how to optimize this (Haskell) program?

```
let f x =
  let z = E3⟨isJust x⟩
  in E0⟨ case x of
    Just a   →  E1⟨a, z⟩
    Nothing  →  E2⟨z⟩ ⟩
in f (Just 2) + f Nothing
```

Problem

Original program:

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After inlining:

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in
(let z0 = E3⟨isJust (Just 2)⟩
    in E0⟨case Just 2 of
      Just a    →  E1⟨a, z0⟩
      Nothing   →  E2⟨z0)⟩)
+
(let z1 = E3⟨isJust (Nothing)⟩
    in E0⟨case Nothing of
      Just a    →  E1⟨a, z1⟩
      Nothing   →  E2⟨z1⟩⟩)
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(let z0 = E3⟨True⟩
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+
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 in E0⟨ E2⟨z1⟩ ⟩)
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Problem: Duplication!

What we would really like:

```
let f0 x0 = E0⟨x0⟩ in let f1 x1 = E3⟨x1⟩ in
f0 (E1⟨2, f1 False⟩) + f0 (E2⟨f1 True⟩)
```

Problems of Inlining

Traditional inlining:

- needs heuristics to avoid code explosion
- causes code duplication (loss of sharing)
- can't handle optimization across recursive calls

Underlying problem: inlining is *all-or-nothing*.

A Graph-Based Approach for Partial/Incremental Inlining

A Graph-Based Approach for Partial Inlining

Ideas:

- Represent functional programs as graphs
- Use special nodes to encode sharing *contexts*
- Adapt the graphs to expose optimizations, without duplicating entire function bodies
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Generalizes several existing optimizations.

Functional Programs as Graphs

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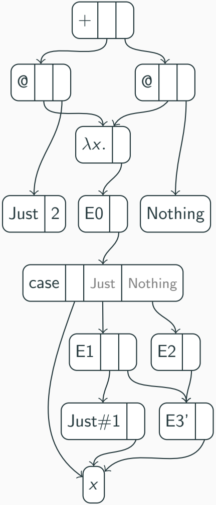
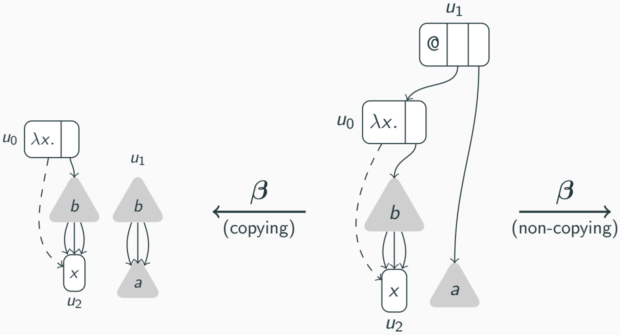


Beta Reduction Without Copying



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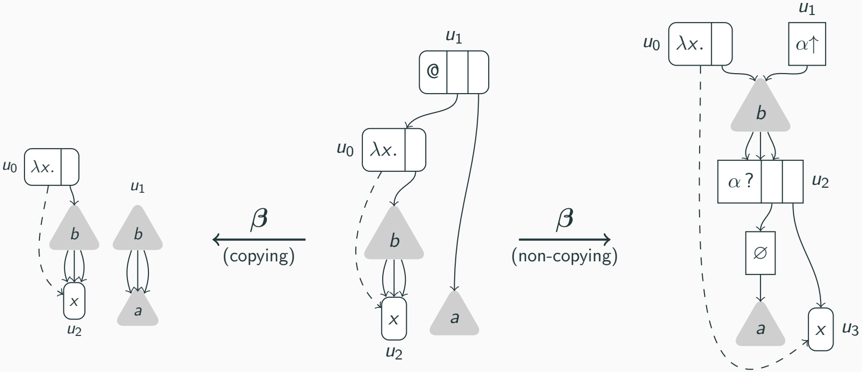
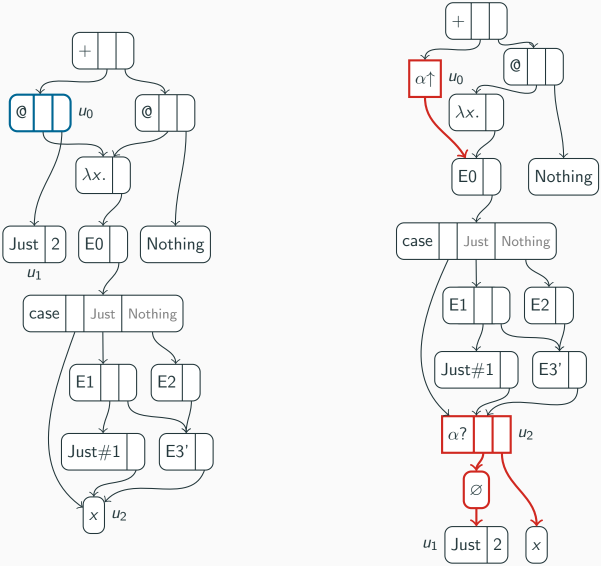
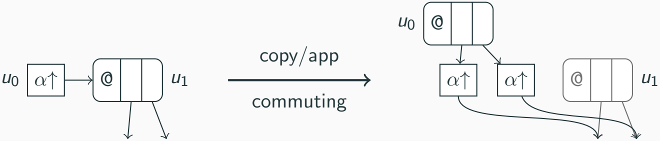


Motivating Example: Beta Reduction



Commuting and Reducing Copy Nodes

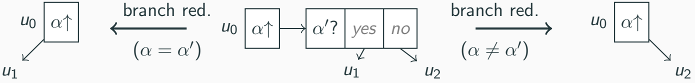
Copying applications



Commuting and Reducing Copy Nodes

Copying applications

Resolving branches



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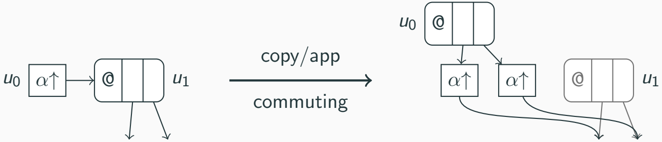

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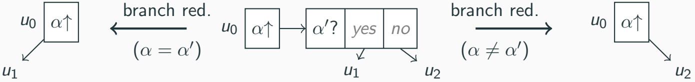
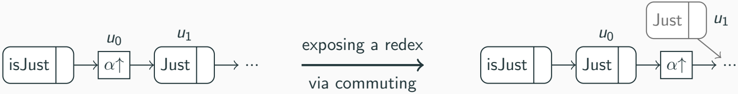


Moreover, copy nodes annihilate with drop nodes: $[\alpha\uparrow]\ [\varnothing]\ u \to u$

Optimizing Across Function Call Boundaries

Pushing copy nodes down:



u_0 u_1

isJust → $\alpha\uparrow$ → Just → ⋯

exposing a redex

via commuting

Just u_1

u_0

isJust → Just → $\alpha\uparrow$ → ⋯

Optimizing Across Function Call Boundaries

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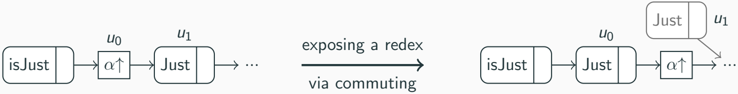
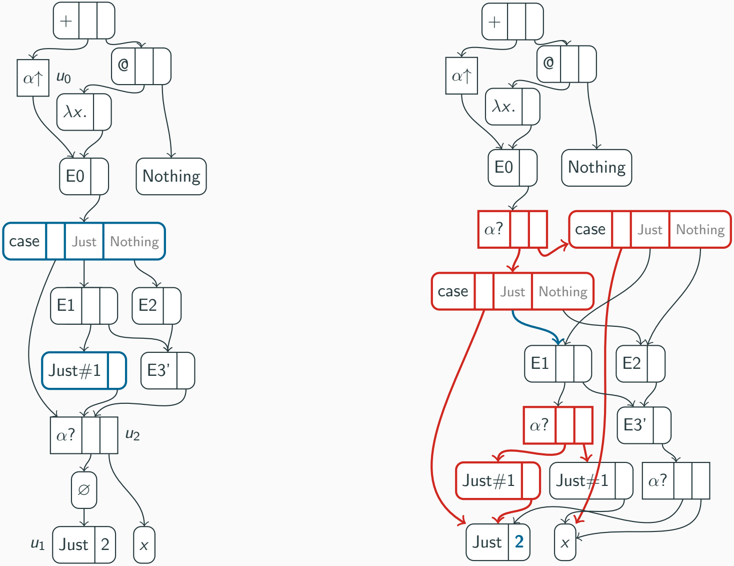
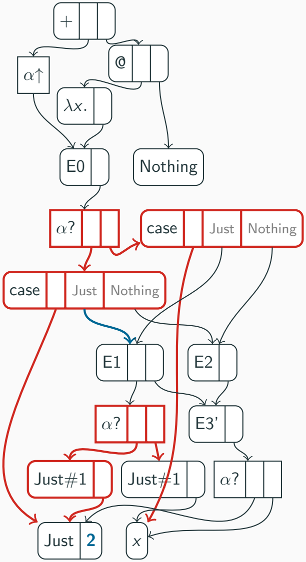


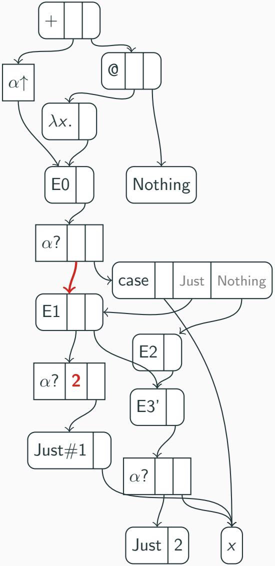
Pulling branch nodes up:

Motivating Example: Commuting



Motivating Example: Reducing





Scopes and Variable Capture

$$f_1 = \lambda x.\ (\lambda y.\ x + y)\ (x + 1)$$

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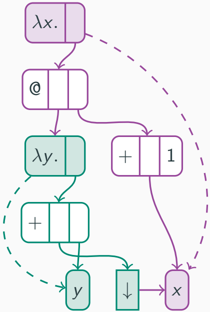

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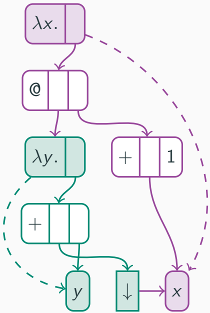

$$f_1 = \lambda x.\ (\lambda y.\ x + y)\ (x + 1)$$

Uses “stop” nodes [↓] to delimit scopes.

More commuting for control nodes

Control nodes $[\,i\,]$ are: “copy” $[\alpha{\uparrow}]$, “drop” $[\varnothing]$, “stop” $[{\downarrow}]$

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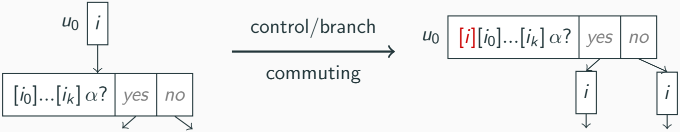

u_0 i

$[i_0]\ldots[i_k]\,\alpha?$ *yes* *no*

control/branch

commuting

u_0 $[i][i_0]\ldots[i_k]\,\alpha?$ *yes* *no*

i i

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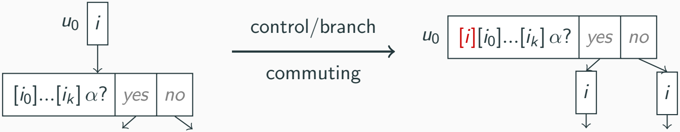

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i i

Copy nodes can be parameterized by a control node instruction *i*:

u_3 $\alpha\uparrow$ i

$\downarrow$

u_4

payload-copy/stop

reduction

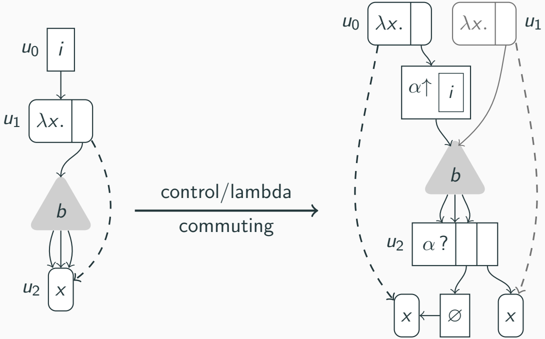
u_3 $\downarrow$

i

u_4

Commuting control noes across lambdas

Instruction parameter introduced when commuting with lambda:



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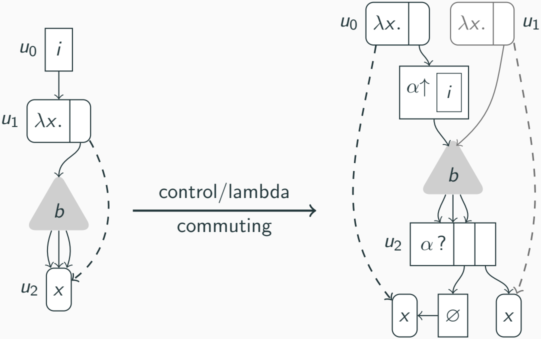

$[\alpha\uparrow[i]]$ releases i when meeting $[\downarrow]$; drops it when meeting $[\varnothing]$.

Properties of PGR

IGR formalized as $\lambda^{\{\mapsto\}}$.

Theorem (Small step rewrites preserve semantics)
Reduction defined in $\lambda^{\{\mapsto\}}$ *is no stronger than strong reduction in* λ *calculus: if* $\mathbf{P}_0 \to \mathbf{P}_1$ *with* $\mathbf{P}_0$ $\mathcal{WS}$*, then* $\mathcal{U}[\![\mathbf{P}_0]\!] \equiv \mathcal{U}[\![\mathbf{P}_1]\!]$.

Theorem (Exhaustiveness of Reduction)
$\mathcal{U}[\![\cdot]\!]$ *is a simulation: if* $\mathcal{U}[\![P_0]\!] \rightharpoonup e_1$ *then there exists a* P_1 *such that* $P_0 \rightharpoonup^* P_1$ *and* $\mathcal{U}[\![P_1]\!] = e_1$.

Theorem (Maximal Sharing)
We do not duplicate applications: in a program's graph after rewriting, there will be at most as many applications as in the original program.

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Incidental result: IGR is a β-optimal evaluator

Scheduling

Ideas:

- each copy identifier denotes a *scope*, in which runtime work is shared
  - copy node: function return
  - drop node: function parameter
  - stop node: variable capture
- reconstitute scopes as corresponding functions
- branches that cannot be solved locally use a flag

  — consider: $[[\varnothing]\ [\varnothing]\,]\alpha?\ldots$
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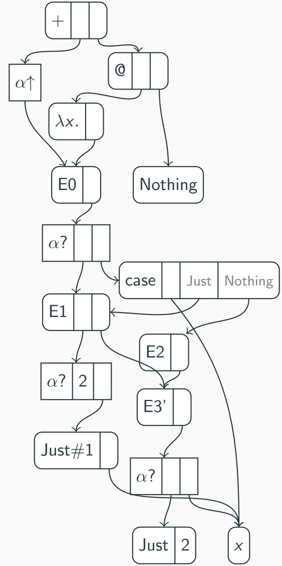
Example: `f a = let tmp = g a in (tmp + 1, tmp - 1)`

with usage: `case f a of (u,v) → u + v`

Motivating Example: Scheduling



After scheduling:

```
let f0 x0 = E0⟨x0⟩ in
let f1 x1 = E3⟨x1⟩ in
f0 (E1⟨2, f1 False⟩)
 + f0 (E2⟨f1 True⟩)
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Enabled Optimizations

Generalized optimization techniques:

- Function outlining, partial inlining
- Uncurrying and efficient multiple returns
- Call-pattern specialisation
- Return-pattern specialisation (new)

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A new approach to:

- Online partial evaluation
- Rewrite rule application
- Handling of join points (immediate or “obvious” in the graph)
- Lambda lifting and defunctionalization
- Deforestation

Uncurrying and efficient multiple returns

After reductions, `P` and `Q` have *equivalent* PGR representations:

```
P: let f  x  y  = x : f  y  x
     in ... f  a  b  ... f  c  d  ...
Q: let f (x, y) = x : f (y, x)
     in ... f (a, b) ... f (c, d) ...
```

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After reductions, `P` and `Q` have *equivalent* PGR representations:

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     in ... f (a, b) ... f (c, d) ...
```

Use the most efficient implementation of argument-passing available — in Haskell, unboxed tuples:

```
let f (# x, y #) = x : f (# y, x #)
 in ... f (# a, b #) ... f (# c, d #) ...
```

Return-pattern Specialisation

Out of the box: optimize across recursive calls:

```
maxMaybe [] = Nothing
maxMaybe (x : xs) = case maxMaybe xs of
  Just m  →  Just (if x > m then x else m)
  Nothing  →  Just x
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Program name	GHC	PGR + GHC
maxMaybe	136.0 (6.176)	33.41 (3.297)

(All optimized with GHC -O3.)

Online Partial Evaluation

Uses recursion markers; allows reducing recursive functions with non-recursive subgraphs

```
max3 x y z = fromJust (maxMaybe [x, y, z])
```

Optimized to:

```
max3 x y z =
 let c = case y > z of {True → y; False → z}
  in case x > c of {True → x; False → c}
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Program name	GHC	PGR + GHC
max3	52.49 (1.039)	29.23 (0.191)

Conclusions

Partial graph reduction (PGR)
makes inlining *not* “all-or-nothing”

Generalizes and facilitates existing optimizations,
making them more robust (no heuristics)

Uses context sharing, similar to optimal reduction

(but cannot be expressed with interaction nets due to some commutings)