Verifying concurrent C programs in Coq

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joint work with Santiago Cuellar, Andrew Appel

Princeton University

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Top to bottom verified sofware development

Verified software development:

- from top: specifications, program logics, static analysers
- to bottom: models of low-level architectures.

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- translations: CompCert certified compiler,
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https://vst.cs.princeton.edu/

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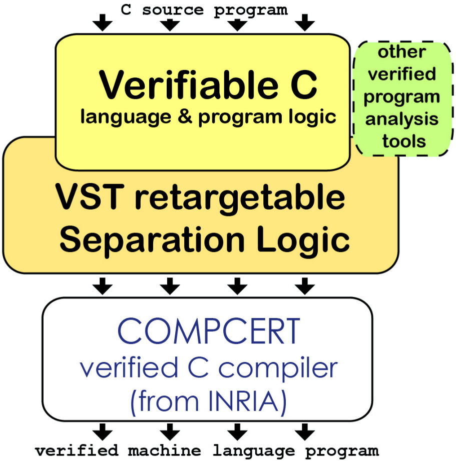
This talk: we try and extend VST to concurrent C programs.

Architecture of VST



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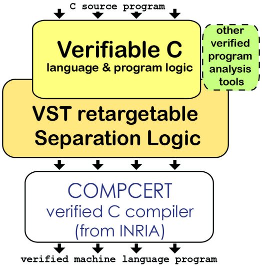

- CompCert : C program $\rightarrow$ Power PC code: preserves the semantics
- VST's separation logic: predicates on this semantics,
- VST's program logic: functional correctness of such programs.

VST's higher-order separation logic

$P ::= \ \ldots$	$v \Downarrow 4$	local variables
	$p \mapsto 4, p \mapsto \_$	pointers, shape
	$f : P \to Q$	function pointers (indirection)
	$!!P$	embedding of Coq propositions
	$\wedge, \vee$	usual logical operators
	$\mu x.P$	recursion
	$\forall x.P, \exists x.P$	impredicative quantification
	$P * Q, P \twoheadrightarrow Q$	non-aliasing (separation)

Recent work was necessary to handle all those features:
Step indexing (Appel, McAllester, TOPLAS 2001)
Step indexing + indirection (Ahmed, Appel, Virga, LICS 2002)
Step indexing + impredicativity (Ahmed PhD thesis 2004)
Very Modal Model (Appel, Melliès, Richards, Vouillon, POPL 2007)
Indirection Theory (Hobor, Dockins, Appel, POPL 2010)

Example of a proof of a program

Example of a C program

```c
struct list {int head; struct list *tail;};

struct list *merge(struct list *a, struct list *b) {
  struct list* ret;
  struct list** x = &ret;
  while (a && b) {
    if (a->head <= b->head) {
      *x = a;
      a = a->tail;
    } else {
      *x = b;
      b = b->tail;
    }
    x = &((*x)->tail);
  }
  *x = (a)?a:b;
  return ret;
}
```

To notice: addressable local variables, pointer to undefined values, loop invariant with partially defined list segments, pointer tricks, no leak.

Same program in **verifiable C**

```c
#include <stddef.h>

struct list {int head; struct list *tail;};

struct list *merge(struct list *a, struct list *b) {
  struct list* ret;
  struct list* temp;
  struct list** x;
  int va, vb, cond;
  x = &ret;
  cond = a != NULL && b != NULL;
  while (cond) {
    va = a->head;
    vb = b->head;
    if (va <= vb) {
      *x = a;
      x = &(a->tail);
      a = a->tail;
    } else {
      *x = b;
      x = &(b->tail);
      b = b->tail;
    }
    cond = a != NULL && b != NULL;
  }
  if (a != NULL) {
    *x = a;
  } else {
    *x = b;
  }
  temp = ret;
  return temp;
}
```

Transformation to *verifiable* C:

- `temp`: addressable variables can't be returned
- `va`, `vb`: tests can be on local expressions only
- `cond`: tests can't be transformed in instructions
- loads and stores must be top-level (which forbids `x = &((*x)->tail);`)

(the transformation could be done automatically, but we still need to reason on this program)

Proof of merge.c

File Edit Options Buffers Tools Coq Proof-General Holes Help

```
  extract_exists_pre] for us. *)
rename a into init_a.
rename b into init_b.
clear a_ b_.
Intros cond a b merged a_ b_ c_ begin.
 forward.


(* The loop *)
forward_while (merge_invariant _cond sh init_a init_b ret_)
  [[[[[[[cond0 a0] b0] merged0] a_0] b_0] c_0] begin0].
+ (* Loop: precondition => invariant *)
 Exists cond a b merged a_ b_ c_ begin; entailer!.
+ (* Loop: condition has nice format *)
now entailer!.
+ (* Loop body preserves invariant *)
clear - SH HRE H1 H2.
rename cond0 into cond, a0 into a, b0 into b, merged0 into merged,
 a_0 into a_, b_0 into b_, c_0 into c_, begin0 into begin.
assert (a_ <> nullval) by intuition.
assert (b_ <> nullval) by intuition.
clear H2.
drop_LOCAL 4%nat; clear cond HRE.
rewrite lseg_unfold.
destruct a as [|va a']; simpl.
  (* [a] cannot be empty *)
  normalize. now intuition.
normalize.
intros a_'.
normalize.
(* Now the command [va = a->head] can proceed *)
rewrite list_cell_field_at.
forward.

rewrite lseg_unfold with (v1:=b_).
destruct b as [|vb b']; simpl.
  (* [b] cannot be empty *)
  normalize; now intuition.
normalize.
intros b_'.
normalize.
clear H2 H3.
```

`-:---  verif_merge.v    33% (234,36)  Git-concurrency  (`

```
H1 : merge init_a init_b = merged ++ merge a b
H2 : cond = Int.zero <-> a_ = nullval \/ b_ = nullval
POSTCONDITION := abbreviate : ret_assert
MORE_COMMANDS := abbreviate : statement
H : a_ <> nullval
H0 : b_ <> nullval
============================
 semax Delta
   (PROP  ()
    LOCAL  (temp _a a_; temp _b b_;
    temp _x
      (if merged
       then ret_
       else field_address (Tstruct _list noattr) [StructField _tail] c_);
    lvar _ret tlist ret_; temp _cond (Vint cond))
    SEP  (`(lseg LS sh (map Vint a) a_ nullval);
    `(lseg LS sh (map Vint b) b_ nullval);
    `(data_at Tsh tlist (if merged then Vundef else begin) ret_);
    `(lseg LS sh (map Vint (butlast merged)) begin c_);
    `(if merged
      then emp
      else data_at sh t_struct_list (Vint (last merged), Vundef) c_)))
   (Ssequence
      (Sset _va
         (Efield
            (Ederef (Etempvar _a (tptr (Tstruct _list noattr)))
               (Tstruct _list noattr)) _head tint)) MORE_COMMANDS)
   POSTCONDITION
```

`U:%%-  *goals*     Bot (36,50)    (Coq Goals -2 vl`

`U:%%-  *response*   All (1,0)     (Coq Response vl`

Concurrent programs

Simple concurrent program

```
x = 0;
y = 0;
x++ || y++;
assert(x + y == 2);
```

Proof of simple concurrent program

$$\{x \mapsto \_ * y \mapsto \_\}$$

```
x = 0;
```

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```
y = 0;
```

$$\{x \mapsto 0 * y \mapsto 0\}$$

```
x++ || y++;
```

$$(*) \quad \{x \mapsto 1 * y \mapsto 1\}$$

```
assert(x + y == 2);
```

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Proof of simple concurrent program

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x = 0;
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y = 0;
```
$$\{x \mapsto 0 * y \mapsto 0\}$$
```
x++ || y++;
```
$$(*) \quad \{x \mapsto 1 * y \mapsto 1\}$$
```
assert(x + y == 2);
```
$$\{x \mapsto 1 * y \mapsto 1\}$$

$$(*) \ \frac{\{x \mapsto 0\}\ \texttt{x++}\ \{x \mapsto 1\} \qquad \{y \mapsto 0\}\ \texttt{y++}\ \{y \mapsto 1\}}{\{x \mapsto 0 * y \mapsto 0\}\ \texttt{x++ || y++}\ \{x \mapsto 1 * y \mapsto 1\}}$$

(no "no interference")

Proof of simple concurrent program

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x = 0; y = 0;
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assert(x + y == 2);
```

The program above is safe.

Proof of simple concurrent program

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x = 0; y = 0;
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```

The program above is safe.

But we have no shared resources.

Threads sharing memory

```
x = 0;
x++ || x++;
assert(x >= 0);
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Threads sharing memory

```
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...race?

Races in CompCert

There are **no** data races in CompCert/VST:

- most experimental logic with races are not proved sound for weakly consistent caches;

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Concurrent variants of CompCert:

- **either** [CompCert TSO, Sewell] racy programs with little ability for the compiler to optimize
- **or** [Compositional CompCert, Appel/Beringer/Stewart/Cuellar] coarse-grain concurrency and optimizing compilation of memory operations

Permissions in CompCert

CompCert memory model, version 2:

$$x \overset{\pi}{\mapsto} 4 \quad \text{rather than} \quad x \mapsto 4$$

$$\pi \ ::= \ \texttt{Freeable} \mid \texttt{Writable} \mid \texttt{Readable} \mid \texttt{Nonempty}$$

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- if a thread has `Freeable`, others have no permissions;
- if a thread has `Writable`, others have at most `Nonempty`;
- if a thread has `Readable`, others have at most `Readable`;
- if a thread has `Nonempty`, others have at most `Writable`.

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- if a thread has `Freeable`, others have no permissions;
- if a thread has `Writable`, others have at most `Nonempty`;
- if a thread has `Readable`, others have at most `Readable`;
- if a thread has `Nonempty`, others have at most `Writable`.

`Nonempty` is "comparable with another non-NULL pointer": in `a == b`, if one of `a` or `b` is a pointer value, then either one of them must be `NULL`, or both must be pointers to allocated objects (`Nonempty` ensures there are no other `Freeable`).

Permissions in VST

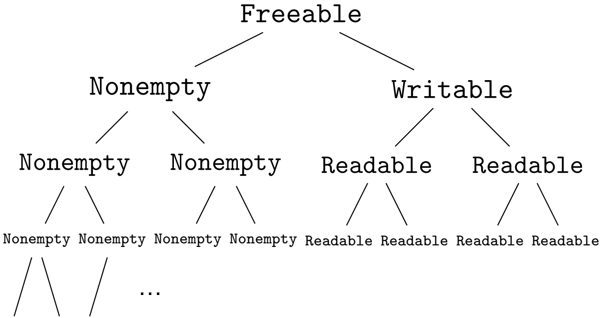
Refinement of CompCert's permissions:

$$\pi \quad ::= \quad \bullet \quad \mid \quad \circ \quad \mid \quad \overset{\wedge}{\pi_1 \ \ \pi_2}$$

Joining permissions:

$$\overset{\wedge}{\bullet \ \ \circ} \oplus \overset{\wedge}{\circ \ \ \bullet} = \overset{\wedge}{\bullet \ \ \bullet} = \bullet \qquad \frac{\pi = \pi_1 \oplus \pi_2}{p \overset{\pi}{\mapsto} v = p \overset{\pi_1}{\mapsto} v * p \overset{\pi_2}{\mapsto} v}$$

Embedding, depending on where the $\bullet$s are:



Threads sharing memory

```
x = 0;
x++ || x++;
assert(x >= 0);
```

...race?

Threads sharing memory, using binary semaphores

```
x = 0;
V(s);
P(s); || P(s);
x++;  || x++;
V(s); || V(s);
P(s);
assert(x >= 0);
```

no race!

Threads sharing memory, using semaphores in Pthreads

```c
#include <pthread.h>
#include <semaphore.h>

void assert(int i) {i = 1/i;}

sem_t s;
int x;

int main (void) {
  pthread_t th;
  x = 0;
  sem_init(&s, 0, 0);
  sem_post(&s);
  pthread_create(&th, NULL, f, (void*)&x);
  sem_wait(&s);
  x++;
  sem_post(&s);
  pthread_join(th, NULL);
  sem_wait(&s);
  sem_destroy(&s);
  assert(x >= 0);
  return 0;
}
```

```c
void* f(void *arg) {
  sem_wait(&s);
  x++;
  sem_post(&s);
  pthread_exit(NULL);
}
```

Threads sharing memory, using binary semaphores

Binary semaphores contain permissions, here on x, which can be transferred between threads:

`P(s);`	`P(s);`
`x++;`	`x++;`
`V(s);`	`V(s);`

Threads sharing memory, using binary semaphores

Binary semaphores contain permissions, here on x, which can be transferred between threads:

$$
\begin{array}{l||l}
\{s \mapsto \mathsf{lock}[x]\} & \{s \mapsto \mathsf{lock}[x]\} \\
\texttt{P(s);} & \texttt{P(s);} \\
\{s \mapsto \mathsf{lock}[x] * x \overset{\bullet}{\mapsto} \_\} & \{s \mapsto \mathsf{lock}[x] * x \overset{\bullet}{\mapsto} \_\} \\
\texttt{x++;} & \texttt{x++;} \\
\{s \mapsto \mathsf{lock}[x] * x \overset{\bullet}{\mapsto} \_\} & \{s \mapsto \mathsf{lock}[x] * x \overset{\bullet}{\mapsto} \_\} \\
\texttt{V(s);} & \texttt{V(s);} \\
\{s \mapsto \mathsf{lock}[x]\} & \{s \mapsto \mathsf{lock}[x]\}
\end{array}
$$

Threads sharing memory

Permissions can be refined to *lock invariants*:

$\{s \mapsto \mathsf{lock}[\exists n \geq 0\ x \overset{\bullet}{\mapsto} n]\}$	$\{s \mapsto \mathsf{lock}[\exists n \geq 0\ x \overset{\bullet}{\mapsto} n]\}$
`P(s);`	`P(s);`
$\{s \mapsto \mathsf{lock}[\exists n \geq 0\ x \overset{\bullet}{\mapsto} n] * \exists n \geq 0\ x \overset{\bullet}{\mapsto} n\}$	$\{s \mapsto \mathsf{lock}[\exists n \geq 0\ x \overset{\bullet}{\mapsto} n] * \exists n \geq 0\ x \overset{\bullet}{\mapsto} n\}$
`x++;`	`x++;`
$\{s \mapsto \mathsf{lock}[\exists n \geq 0\ x \overset{\bullet}{\mapsto} n] * \exists n \geq 0\ x \overset{\bullet}{\mapsto} n\}$	$\{s \mapsto \mathsf{lock}[\exists n \geq 0\ x \overset{\bullet}{\mapsto} n] * \exists n \geq 0\ x \overset{\bullet}{\mapsto} n\}$
`V(s);`	`V(s);`
$\{s \mapsto \mathsf{lock}[\exists n \geq 0\ x \overset{\bullet}{\mapsto} n]\}$	$\{s \mapsto \mathsf{lock}[\exists n \geq 0\ x \overset{\bullet}{\mapsto} n]\}$

Threads sharing memory

Atomicity is not mandatory:

$\{s \mapsto \mathsf{lock}[\exists n \geq 0\ x \overset{\bullet}{\mapsto} n]\}$	$\{s \mapsto \mathsf{lock}[\exists n \geq 0\ x \overset{\bullet}{\mapsto} n]\}$
`P(s);`	`P(s);`
$\{s \mapsto \mathsf{lock}[\exists n \geq 0\ x \overset{\bullet}{\mapsto} n] * \exists n \geq 0\ x \overset{\bullet}{\mapsto} n\}$	$\{s \mapsto \mathsf{lock}[\exists n \geq 0\ x \overset{\bullet}{\mapsto} n] * \exists n \geq 0\ x \overset{\bullet}{\mapsto} n\}$
`a = x;`	`a = x;`
`x = a - 2; // x can be negative here`	`x = a - 3;`
`x = a + 1;`	`x = a + 1;`
$\{s \mapsto \mathsf{lock}[\exists n \geq 0\ x \overset{\bullet}{\mapsto} n] * \exists n \geq 0\ x \overset{\bullet}{\mapsto} n\}$	$\{s \mapsto \mathsf{lock}[\exists n \geq 0\ x \overset{\bullet}{\mapsto} n] * \exists n \geq 0\ x \overset{\bullet}{\mapsto} n\}$
`V(s);`	`V(s);`
$\{s \mapsto \mathsf{lock}[\exists n \geq 0\ x \overset{\bullet}{\mapsto} n]\}$	$\{s \mapsto \mathsf{lock}[\exists n \geq 0\ x \overset{\bullet}{\mapsto} n]\}$

Threads sharing memory

Coming back to `x++`:

$\{s \mapsto \mathsf{lock}[\exists n \geq 0\ x \overset{\bullet}{\mapsto} n]\}$	$\{s \mapsto \mathsf{lock}[\exists n \geq 0\ x \overset{\bullet}{\mapsto} n]\}$
`P(s);`	`P(s);`
$\{s \mapsto \mathsf{lock}[\exists n \geq 0\ x \overset{\bullet}{\mapsto} n] * \exists n \geq 0\ x \overset{\bullet}{\mapsto} n\}$	$\{s \mapsto \mathsf{lock}[\exists n \geq 0\ x \overset{\bullet}{\mapsto} n] * \exists n \geq 0\ x \overset{\bullet}{\mapsto} n\}$
`x++;`	`x++;`
$\{s \mapsto \mathsf{lock}[\exists n \geq 0\ x \overset{\bullet}{\mapsto} n] * \exists n \geq 0\ x \overset{\bullet}{\mapsto} n\}$	$\{s \mapsto \mathsf{lock}[\exists n \geq 0\ x \overset{\bullet}{\mapsto} n] * \exists n \geq 0\ x \overset{\bullet}{\mapsto} n\}$
`V(s);`	`V(s);`
$\{s \mapsto \mathsf{lock}[\exists n \geq 0\ x \overset{\bullet}{\mapsto} n]\}$	$\{s \mapsto \mathsf{lock}[\exists n \geq 0\ x \overset{\bullet}{\mapsto} n]\}$

Threads sharing memory

$\{s \mapsto \mathsf{lock}[\exists n \geq 0\ x \mapsto n] * x \stackrel{\bullet}{\mapsto} \_\}$

`x = 0;`

$\{s \mapsto \mathsf{lock}[\exists n \geq 0\ x \stackrel{\bullet}{\mapsto} n] * x \stackrel{\bullet}{\mapsto} 0\}$

`V(s);`

$\{s \mapsto \mathsf{lock}[\exists n \geq 0\ x \stackrel{\bullet}{\mapsto} n]\}$

`(P(s); x++; V(s)) || (P(s); x++; V(s));`

$\{s \mapsto \mathsf{lock}[\exists n \geq 0\ x \stackrel{\bullet}{\mapsto} n]\}$

`P(s);`

$\{s \mapsto \mathsf{lock}[\exists n \geq 0\ x \stackrel{\bullet}{\mapsto} n] * \exists n \geq 0\ x \mapsto n\}$

`assert(x >= 0);`

$\{s \mapsto \mathsf{lock}[\exists n \geq 0\ x \stackrel{\bullet}{\mapsto} n] * \exists n \geq 0\ x \mapsto n\}$

Threads sharing memory

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The above program is safe.

Threads sharing memory

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The above program is safe.

Threads sharing memory

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P(s);
assert(x >= 0);
```

The above program is safe.

But we can know more about `x`

Threads sharing memory

```
x = 0;
V(s);
(P(s); x++; V(s)) || (P(s); x++; V(s));
P(s);
assert(x == 2);
```

Threads sharing memory

```
x = 0;
V(s);
(P(s); x++; V(s)) || (P(s); x++; V(s));
P(s);
assert(x == 2);
```

Invariants are not enough.

Threads sharing memory: ghost variables

```
x = 0; x1 = 0; x2 = 0;
V(s);
(P(s); x1++; x++; V(s)) || (P(s); x2++; x++; V(s));
P(s);
assert(x == 2);
```

Ghost variables

A new invariant relying on ghost variables: $R = \exists n_1, n_2 \begin{array}{l} x \overset{\bullet}{\mapsto} n_1 + n_2 \\ * \; x_1 \overset{\LEFTcircle}{\mapsto} n_1 \\ * \; x_2 \overset{\LEFTcircle}{\mapsto} n_2 \end{array}$

$\{l \boxdot\!\!\overset{\LEFTcircle}{\mapsto} R * x_2 \overset{\LEFTcircle}{\mapsto} 0\}$

```
P(l);
```

$\{l \boxdot\!\!\overset{\LEFTcircle}{\mapsto} R * \exists n_1 \; x_2 \overset{\bullet}{\mapsto} 0 * x \overset{\bullet}{\mapsto} n_1 + 0 * x_1 \overset{\LEFTcircle}{\mapsto} n_1\}$

$\{l \boxdot\!\!\overset{\LEFTcircle}{\mapsto} R * x_2 \overset{\bullet}{\mapsto} 0 * x \overset{\bullet}{\mapsto} n_1 + 0 * x_1 \overset{\LEFTcircle}{\mapsto} n_1\}$

```
x2++;
```

$\{l \boxdot\!\!\overset{\LEFTcircle}{\mapsto} R * x_2 \overset{\bullet}{\mapsto} 1 * x \overset{\bullet}{\mapsto} n_1 + 0 * x_1 \overset{\LEFTcircle}{\mapsto} n_1\}$

```
x++;
```

$\{l \boxdot\!\!\overset{\LEFTcircle}{\mapsto} R * x_2 \overset{\bullet}{\mapsto} 1 * x \overset{\bullet}{\mapsto} n_1 + 1 * x_1 \overset{\LEFTcircle}{\mapsto} n_1\}$

$\{l \boxdot\!\!\overset{\LEFTcircle}{\mapsto} R * x_2 \overset{\LEFTcircle}{\mapsto} 1 * R\}$

```
V(l);
```

$\{l \boxdot\!\!\overset{\LEFTcircle}{\mapsto} R * x_2 \overset{\LEFTcircle}{\mapsto} 1\}$

Threads sharing memory

$\{s \overset{\bullet}{\boxdot\!\!\rightarrow} R * x \overset{\bullet}{\mapsto} \_\}$

`x = 0; x1 = 0; x2 = 0;`

$\{s \overset{\bullet}{\boxdot\!\!\rightarrow} R * x \overset{\bullet}{\mapsto} 0 * x_1 \overset{\bullet}{\mapsto} 0 * x_2 \overset{\bullet}{\mapsto} 0\}$

`V(s);`

$\{s \overset{\bullet}{\boxdot\!\!\rightarrow} R * x_1 \overset{\circ}{\mapsto} 0 * x_2 \overset{\circ}{\mapsto} 0\}$

`(P(s); x1++; x++; V(s)) || (P(s); x2++; x++; V(s));`

$\{s \overset{\bullet}{\boxdot\!\!\rightarrow} R * x_1 \overset{\circ}{\mapsto} 1 * x_2 \overset{\circ}{\mapsto} 1\}$

`P(s);`

$\{s \overset{\bullet}{\boxdot\!\!\rightarrow} R * x \overset{\bullet}{\mapsto} 2 * x_1 \overset{\bullet}{\mapsto} 1 * x_2 \overset{\bullet}{\mapsto} 1\}$

`assert(x == 2);`

$\{s \overset{\bullet}{\boxdot\!\!\rightarrow} R * x \overset{\bullet}{\mapsto} 2 * x_1 \overset{\bullet}{\mapsto} 1 * x_2 \overset{\bullet}{\mapsto} 1\}$

Threads sharing memory

$\{s \boxdot\!\!\overset{\bullet}{\mapsto} R * x \overset{\bullet}{\mapsto} \_\}$

`x = 0; x1 = 0; x2 = 0;`

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`V(s);`

$\{s \boxdot\!\!\overset{\bullet}{\mapsto} R * x_1 \overset{\circ}{\mapsto} 0 * x_2 \overset{\circ}{\mapsto} 0\}$

`(P(s); x1++; x++; V(s)) || (P(s); x2++; x++; V(s));`

$\{s \boxdot\!\!\overset{\bullet}{\mapsto} R * x_1 \overset{\circ}{\mapsto} 1 * x_2 \overset{\circ}{\mapsto} 1\}$

`P(s);`

$\{s \boxdot\!\!\overset{\bullet}{\mapsto} R * x \overset{\bullet}{\mapsto} 2 * x_1 \overset{\bullet}{\mapsto} 1 * x_2 \overset{\bullet}{\mapsto} 1\}$

`assert(x == 2);`

$\{s \boxdot\!\!\overset{\bullet}{\mapsto} R * x \overset{\bullet}{\mapsto} 2 * x_1 \overset{\bullet}{\mapsto} 1 * x_2 \overset{\bullet}{\mapsto} 1\}$

The above program is safe.

But

Problems:

- unbounded number of ghost variables?

But

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- unbounded number of ghost variables, or thread flow unknown?

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Solution:

- we use a *enriched* memory (same as for $f : \{P\} \to \{Q\}$):

$$\frac{\{\exists g\ g \overset{\bullet}{\mapsto} v * P\}\ c\ \{Q\}}{\{P\}\ c\ \{Q\}} \qquad \frac{\{g \overset{\bullet}{\mapsto} v' * P\}\ c\ \{Q\}}{\{g \overset{\bullet}{\mapsto} v * P\}\ c\ \{Q\}}$$

$$g \overset{\bullet}{\mapsto} v \ = \ g \overset{◐}{\mapsto} v \ * \ g \overset{◑}{\mapsto} v$$

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$$g \overset{\bullet}{\mapsto} v \ = \ g \overset{\LEFTcircle}{\mapsto} v \ * \ g \overset{\RIGHTcircle}{\mapsto} v$$

- Importantly, when we own $g \overset{\LEFTcircle}{\mapsto} v$ or $\exists v\ g \overset{\LEFTcircle}{\mapsto} v$ we know that v is not modified by another thread.

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- **semantic** erasure
- infinite number of ghost variables? **indexed** g_i**'s**?
  How to organise them? (we must keep an infinite supply!)

Splitting infinite sets

We can split infinite subsets, e.g. for $\mathbb{N}$:

$$\mathbb{N} = (1 + 2\mathbb{N}) \uplus 2\mathbb{N}$$

and more that once:

$$\mathbb{N} = \biguplus_{k \in \mathbb{N}} 2^k(1 + 2\mathbb{N}) - 1$$

We have encountered this problem before!

Permissions shares have been implemented by $z, 0 \leq z \leq 1$, intervals of $[0, 1]$, subsets of $\mathbb{N}$, ... and finally, trees!

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$$t ::= \bullet \mid \circ \mid \genfrac{}{}{0pt}{}{\wedge}{t_1 \quad t_2} \qquad / \qquad \genfrac{}{}{0pt}{}{\wedge}{\bullet \quad \bullet} \equiv \bullet, \ \genfrac{}{}{0pt}{}{\wedge}{\circ \quad \circ} \equiv \circ$$

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Embedding in infinite-or-empty subsets of $\mathbb{N}$:

$$\mathbb{N}_\bullet = \mathbb{N} \qquad \mathbb{N}_\circ = \emptyset \qquad \mathbb{N}_{\left(\overset{\wedge}{t_1 \quad t_2}\right)} = 2\mathbb{N}_{t_1} \uplus (1 + 2\mathbb{N}_{t_2})$$

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Converse ("terminates" on $[\![\cdot]\!]$; $f \circ \mathbb{N}.$ is the normalization function for $\equiv$)

$$f(\emptyset) = \circ \qquad f(\mathbb{N}) = \bullet$$

$$f(A) = \overset{\wedge}{t_1 \quad t_2} \text{ with } t_1 = f\left(\frac{A \cap 2\mathbb{N}}{2}\right) \text{ and } t_2 = f\left(\frac{A \cap (1 + 2\mathbb{N}) - 1}{2}\right)$$

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These $\mathbb{N}_t$ help us embed our ghost state in our memory model.

Representation of ghost state

$$g \overset{\pi}{\underset{\rho}{\mapsto}} v \quad \triangleq \quad \exists (v_i) \prod_{i \in \mathbb{N}_\rho} g_i \overset{\pi}{\mapsto} v_i \wedge \sum_{i \in \mathbb{N}_\rho} v_i = v$$

Representation of ghost state

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Two tree shares:

- π: permission (what can we do...)
- ρ: location (...to which part)

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$$\frac{\pi_1 \oplus \pi_2 = \pi}{g \overset{\pi_1}{\underset{\rho}{\mapsto}} v * g \overset{\pi_2}{\underset{\rho}{\mapsto}} v = g \overset{\pi}{\underset{\rho}{\mapsto}} v}$$

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Threads sharing memory

$$\{s \overset{\bullet}{\boxdot\!\!\rightarrow} R * x \overset{\bullet}{\mapsto} 0\}$$

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```
V(s);
```

Threads sharing memory

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`V(s);`

$R \triangleq \exists v\ x \overset{\bullet}{\mapsto} v * g \underset{\bullet}{\overset{\circ}{\mapsto}} v$

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Threads sharing memory

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```
(P(s); x++; V(s)) || (P(s); x++; V(s));
```

Threads sharing memory

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$\{s \overset{\bullet}{\boxdot\!\!\rightarrow} R * g \overset{\LEFTcircle}{\underset{\bullet}{\mapsto}} 2\}$

Threads sharing memory

$\{s \overset{\bullet}{\boxdot\!\!\rightarrow} R * x \overset{\bullet}{\mapsto} 0\}$

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(P(s); x++; V(s)) || (P(s); x++; V(s));
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$\{s \overset{\bullet}{\boxdot\!\!\rightarrow} R * g \underset{\LEFTcircle}{\overset{\LEFTcircle}{\mapsto}} 1 * g \underset{\RIGHTcircle}{\overset{\LEFTcircle}{\mapsto}} 1\}$

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$\{s \overset{\bullet}{\boxdot\!\!\rightarrow} R * x \overset{\bullet}{\mapsto} 2 * g \underset{\bullet}{\overset{\LEFTcircle}{\mapsto}} 2\}$

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assert(x == 2);
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Threads sharing memory

$\{s \boxdot\!\!\overset{\bullet}{\mapsto} R * x \overset{\bullet}{\mapsto} 0\}$
$\{s \boxdot\!\!\overset{\bullet}{\mapsto} R * x \overset{\bullet}{\mapsto} 0 * g \overset{\bullet}{\underset{\bullet}{\mapsto}} 0\}$

`V(s);` $\qquad R \triangleq \exists v\ x \overset{\bullet}{\mapsto} v * g \overset{\RIGHTcircle}{\underset{\bullet}{\mapsto}} v$

$\{s \boxdot\!\!\overset{\bullet}{\mapsto} R * g \overset{\LEFTcircle}{\underset{\bullet}{\mapsto}} 0\}$

$\{s \boxdot\!\!\overset{\bullet}{\mapsto} R * g \overset{\LEFTcircle}{\underset{\LEFTcircle}{\mapsto}} 0 * g \overset{\LEFTcircle}{\underset{\RIGHTcircle}{\mapsto}} 0\}$

```
(P(s); x++; V(s)) || (P(s); x++; V(s));
```

$\{s \boxdot\!\!\overset{\bullet}{\mapsto} R * g \overset{\LEFTcircle}{\underset{\LEFTcircle}{\mapsto}} 1 * g \overset{\LEFTcircle}{\underset{\RIGHTcircle}{\mapsto}} 1\}$

$\{s \boxdot\!\!\overset{\bullet}{\mapsto} R * g \overset{\LEFTcircle}{\underset{\bullet}{\mapsto}} 2\}$

`P(s);`

$\{s \boxdot\!\!\overset{\bullet}{\mapsto} R * x \overset{\bullet}{\mapsto} 2 * g \overset{\LEFTcircle}{\underset{\bullet}{\mapsto}} 2\}$

$\{s \boxdot\!\!\overset{\bullet}{\mapsto} R * x \overset{\bullet}{\mapsto} 2\}$

`assert(x == 2);`

$\{s \boxdot\!\!\overset{\bullet}{\mapsto} R * x \overset{\bullet}{\mapsto} 2\}$

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V(s);
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safe!

... we catch back on Nanevski's subjective views

Thread 1 | Shared resource | Thread 2

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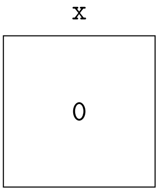
x = 0;
```



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x = 0;
V(s);
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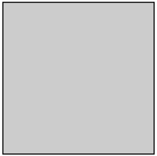
x



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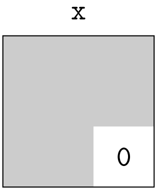


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Shared resource

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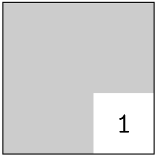

Thread 2

```
P(s);
x++;
```

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```

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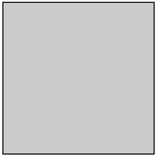

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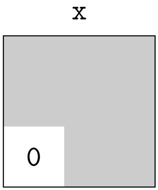

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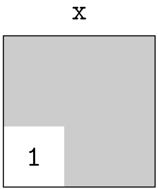

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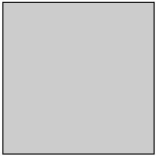

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P(s);
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Shared resource

x

0
$\oplus$
1 1

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... we catch back on Nanevski's subjective views

Thread 1	Shared resource	Thread 2
`x = 0;` `V(s);`		
	x $\boxed{2}$	

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`P(s);` `assert(x == 2);`		

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Thread 1 | Shared resource | Thread 2

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```
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V(s);

P(s);
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V(s);

P(s);
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Shared resource:

x

$\boxed{2}$

Thread 2:

```
P(s);
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Safe!

Summary

```c
#include <pthread.h>
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void assert(int i) {i = 1/i;}

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void* f(void *arg) {
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   $\simeq$ our ghost state

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We don't do:

- RCU,
- races, low-level barriers,
- lock-free implementations.

Thank you for having me!

$$g \xmapsto[\rho]{\pi} v$$

https://github.com/PrincetonUniversity/VST/tree/concurrency