#### **CUTECat**

Concolic Execution for Computational Law

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### Computational law

- · Computational laws encode algorithms: taxes, social benefits, etc.
- Administrations implement them as programs
- Critical: *e.g.* French military payroll system Louvois: 120k military personnel over- or under-paid, overpayments totalling 545M € to pay back

#### Article 1

The income tax is a fixed percentage of the income.

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#### Article 3

If the income is less than \$10,000, the percentage mentioned at article 1 is 10%.

#### Article 4

For people in charge of 3 or more children, the percentage mentioned at article 1 is 15%.

Article 1

Article 3

exception

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Article 2

default case

Article 4

exception

The fixed percentage mentioned at article 1 is 20%.

For people in charge of 3 or more children, the percentage mentioned at article 1 is 15%.

Default logic

## The Catala domain-specific language

```
# Article 3
# Article 1
The income tax is a fixed percentage of
                                           If the income is less than $10,000, the
the income.
                                           percentage mentioned at article 1 is 10%.
                                              catala
scope IncomeTaxComputation:
                                           scope IncomeTaxComputation:
definition income tax equals
                                            exception definition tax rate
                                             under condition house.income <= $10,000</pre>
                                             consequence equals 10%
# Article 2
The fixed percentage mentioned at
                                           # Article 4
article 1 is 20%.
                                           For people in charge of 3 or more
                                           children, the percentage mentioned at
                                           article 1 is 15%.
scope IncomeTaxComputation:
  definition tax rate equals 20%
                                           scope IncomeTaxComputation:
                                            exception definition tax rate
                                             under condition house.nb children >= 3
                                             consequence equals 15%
   · Literate programming
```

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### The Catala domain-specific language

#### # Article 1

The income tax is a fixed percentage of the income.

scope IncomeTaxComputation:
 definition income\_tax equals
 house.income \* tax\_rate

#### # Article 2

The fixed percentage mentioned at article 1 is 20%.

scope IncomeTaxComputation:
 definition tax\_rate equals 20%

· Literate programming

#### # Article 3

If the income is less than \$10,000, the percentage mentioned at article 1 is 10%.

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 under condition house.income <= \$10,000
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For people in charge of 3 or more children, the percentage mentioned at article 1 is 15%.

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## The Catala domain-specific language

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4/22

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• Follows the exception/default structure of the law

#### Kinds of error

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  - interpretation conflicts, e.g. income = \$9,000 and children = 4
  - unhandled cases

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- Ambiguities in the code
  - interpretation conflicts, e.g. income = \$9,000 and children = 4
  - unhandled cases
  - in Catala: ambiguity = runtime error
  - resolved by lawyer/court if implementation is correct
- Other errors: division by zero, assertion error, etc.

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- Systematically:
  - find complex corner cases
  - complete coverage
- · Handle default logic
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    - no loops or memory
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    - no loops or memory
    - · all programs terminate
  - · But some features are hard to encode symbolically

#### Outline

Concolic execution of default terms

Performance and usability improvements

Experimental evaluation

# Concolic execution of default terms

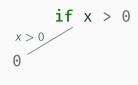
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if x > 0
    then 0
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    then 1
    else error</pre>
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Concolic = concrete + symbolic

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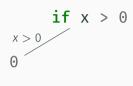
Step | x y Output Constraints after evaluation Next path to try

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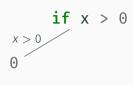
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1   1	20	0	x > 0	

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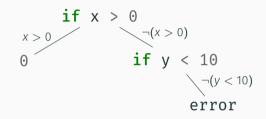


Step   x	У	Output	Constraints after evaluation	Next path to try	
1   1	20	0	<i>x</i> > 0	$\neg(x>0)$	_ ⊋ S.

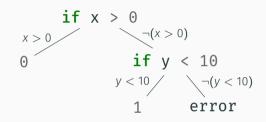
⊃ SMT



Step	X	У	Output	Constraints after evaluation	Next path to try	
1	1	20	0	x > 0	$\neg (x > 0)$	) SMT
2	0	20				



Step	X	У	Output	Constraints after evaluation	Next path to try	
1	1	20	0	<i>x</i> > 0	$\neg(x>0)$	SMT
2	0	20	error	$x > 0$ $\neg(x > 0) \land \neg(y < 10)$	$\neg(x>0) \land y<10$	⇒ SMT



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1	1	20	0	x > 0	$\neg(x>0)$	⊃ SMT
				$\neg(x>0) \land \neg(y<10)$	$\neg(x>0) \land y<10$	
3	0	9	1	$\neg(x>0) \land y<10$	-	*

Source code 
$$\xrightarrow{\text{compiler}}$$
 default terms

$$e ::= \langle e_1, \ldots, e_n \mid b_{default} :- e_{default} \rangle$$

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$$e := \langle \underbrace{e_1, \dots, e_n}_{\text{exceptions}} \mid b_{\text{default}} : - e_{\text{default}} \rangle$$

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$$v ::= \underbrace{\text{true}}_{\text{exceptions}} \mid \text{false} \mid n \mid \dots$$

$$\mid \varnothing$$

$$\mid \mathscr{Q}$$

$$\mid \circledast$$

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    ⟨ | income ≤ $10,000 :- 10%⟩,
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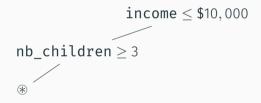
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 $income \leq $10,000$ 

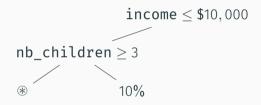
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$$\begin{array}{c} \text{income} \leq \$10,000 \\ \\ \text{nb\_children} \geq 3 \end{array}$$

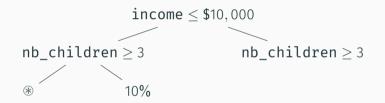
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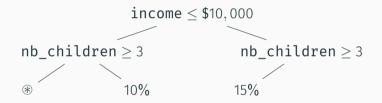
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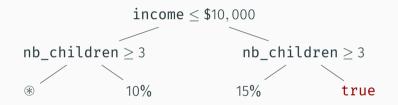
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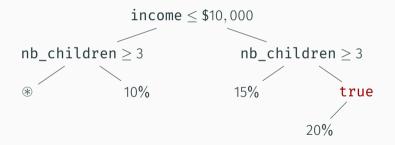
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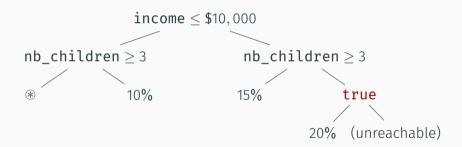
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```



```
\langle\langle | income \leq $10,000: -10%\rangle, \langle | nb_children \geq 3: -15%\rangle | true: -20%\rangle income = ???; nb_children = ???
```



Why not purely symbolic execution?

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- Dates can be ambiguous: what is 29 February 2024 + 1 year?
- We want to generate counter-examples

# Fixing the interpretation conflict

Suppose the lawyer says the **income** condition has priority.

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# Fixing the interpretation conflict

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 $\rightarrow$  it becomes an exception to the exception.

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# Performance and usability improvements

## Performance optimizations using reordering

Theorem (Independence of exception evaluation order) If there is a default value v such that

$$\langle ..., e_i, ..., e_j, ... \mid b_{default} : -e_{default} \rangle \longrightarrow^* V,$$

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Example:

$$\langle ..., \circledast \mid b_{default} : -e_{default} \rangle \sim \langle \circledast , ... \mid b_{default} : -e_{default} \rangle$$

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    catala
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- Difficult for lawyers to compute by hand
- Find more usable input values using soft constraints
  - e.g. round to \$1,000

Incremental mode

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  - Solver keeps a stack of constraints

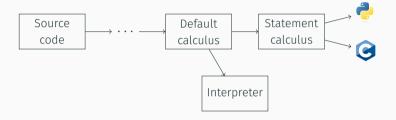
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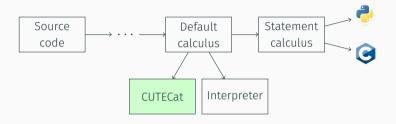
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- · Redundant constraint elimination

# Implementation of CUTECat



## Implementation of CUTECat



- · Integrated into Catala compiler's default calculus IR
- · 3.4k lines of OCaml code
- · Z3 SMT Solver

Experimental evaluation

## Code base

Law	Lines of law in Markdown	Lines of Catala	Total
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French housing benefits	5736	8615	14351
US Tax code § 132	35	56	91
Minimum wage	74	161	235
Family quotient	36	165	201
Handwritten unit tests	139	699	838

# Performance on small programs

Time (s)

Law	No optimizations	Incremental	All opt.	Generated tests
US Tax code	0.27	0.02	0.02	10
Minimum wage	1.01	0.08	0.08	17
Family quotient	82.61	5.21	4.34	381

# Case study: housing benefits

### Key results

- 186,390 test cases generated in **7h of CPU time**
- 99.83% of tests satisfy soft constraints
- · 366s spent in solver, the rest in evaluation
- Able to find a conflict

## Overhead of the analysis

- 4.5x overhead w.r.t. Catala interpreter
- Optimizations make SymCC<sup>1</sup> or SYMSAN<sup>2</sup> reach the same order of magnitude
- KLEE sometimes reports several orders of magnitude<sup>3</sup>

<sup>&</sup>lt;sup>1</sup>Poeplau and Francillon [2020]

<sup>&</sup>lt;sup>2</sup>Chen et al. [2022]

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- Future work: more optimizations

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### Conclusion

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- Novel concolic semantics for default logic
- · Integrated with Catala toolchain
- Optimizations improve efficiency and usability by lawyers
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- Conformance testing
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Contact, ESOP'25 preprint, slides: pierregoutagny.fr

#### References i

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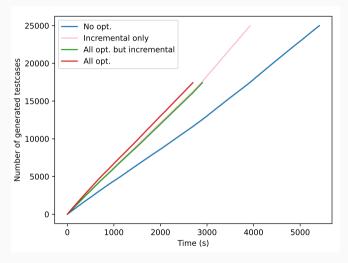
Poeplau, S., Francillon, A.: Symbolic execution with SymCC: Don't interpret, compile! In: Proceedings of the 29th USENIX Conference on Security Symposium, pp. 181–198, SEC'20, USENIX Association, USA (2020), URL https://dl.acm.org/doi/10.5555/3489212.3489223

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https://dl.acm.org/doi/10.5555/3277203.3277260

# Ablation study



Generated tests vs time