

COMPILING PATTERN MATCHING TO IN-PLACE MODIFICATIONS



Paul IANNETTA^{*}, Laure GONNORD[†], Gabriel RADANNE[‡]

^{*} ENS de Lyon, Inria & LIP

[†] UGA, Grenoble INP, LCIS & LIP

[‡] Inria & LIP

paul.iannetta@ens-lyon.fr

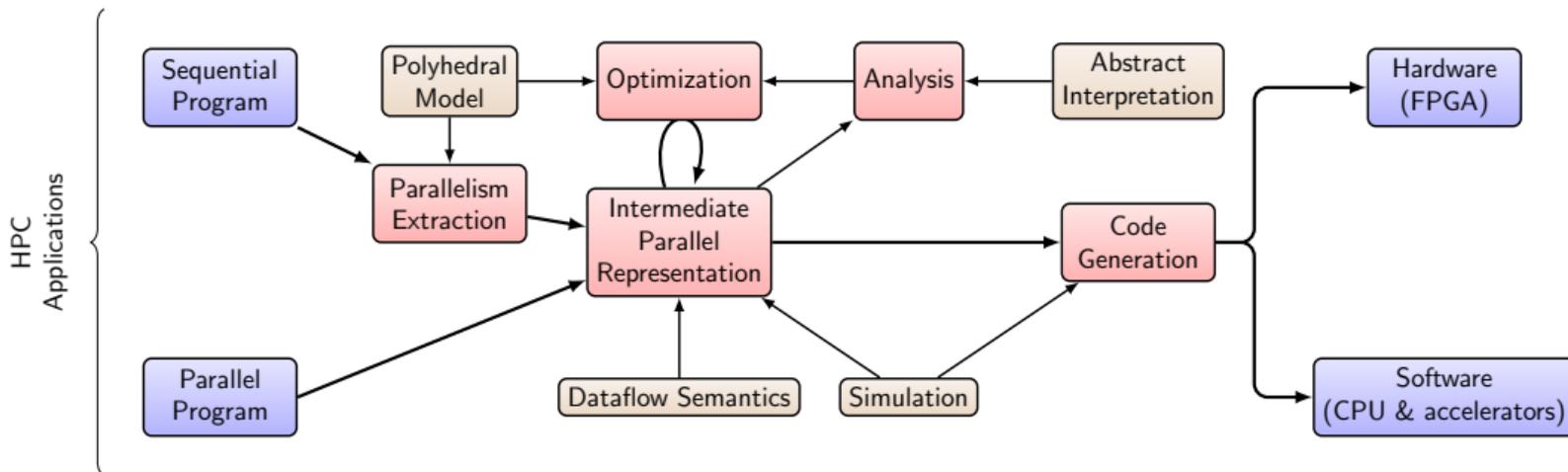
laure.gonnord@lcis.grenoble-inp.fr

gabriel.radanne@inria.fr

CASH: Topics

Optimized (software/hardware) compilation for HPC software with data-intensive computations.

Means: dataflow IR, static analyses, optimisations, simulation.



<http://www.ens-lyon.fr/LIP/CASH/>

Our Starting Point

```
struct tree {
    int a;
    struct tree *l, *r;
};

void mirror_left (struct tree *t) {
    t && t->right = t->left;
}
```

- + Performance
- + Flexibility
- Manual memory handling

```
type tree =
| Empty
| Node of tree * int * tree

let mirror_left = function
| Empty -> Empty
| Node(l,o,r) -> Node(l,o,l)
```

- + Immutable
- Immutable
- Fix Memory Layout

Our Starting Point

st

};

vo

}

How to take the best of both worlds?

- Keep the DSL
- Keep a mutable semantics

+ Flexibility

– Manual memory handling

– Immutable

– Fix Memory Layout

Contents

- 1 **Presentation of REW**
 - Structural Transformations
 - Coarse Memory Movements
 - Fine-Grain Memory Movements
- 2 **Scheduling Memory Movements**
 - Memory Movements Domains
 - Inter-Movement Dependencies
- 3 **Code Generation**
 - Schedule Computation
 - Application of the Algorithm to the Running Example
- 4 **Some Manual Experiments**
 - AVL Trees
 - Decomposition of Rotations
 - Benchmarks

What is REW?

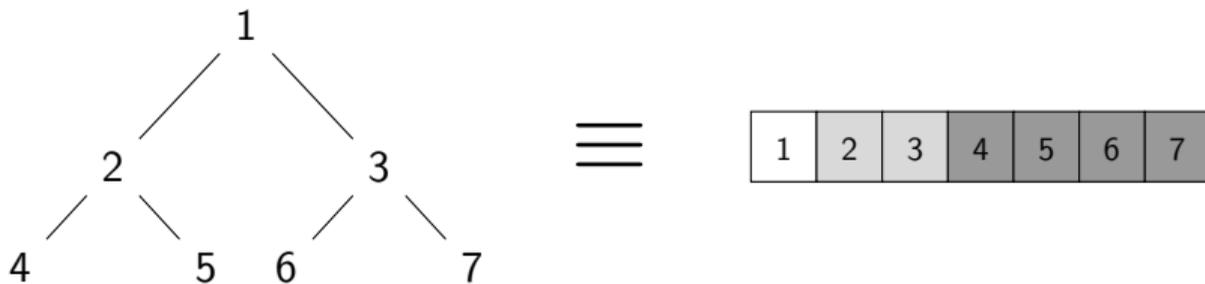
REW is small DSL to:

- declare Algebraic Data Types with:
 - no sharing
 - no mutual recursion
 - all constructors slots must have a bounded size at compilation time
 - all constructors are of finite arity
- describe structural transformations on those through pattern matching.

Running Example: Pull Up

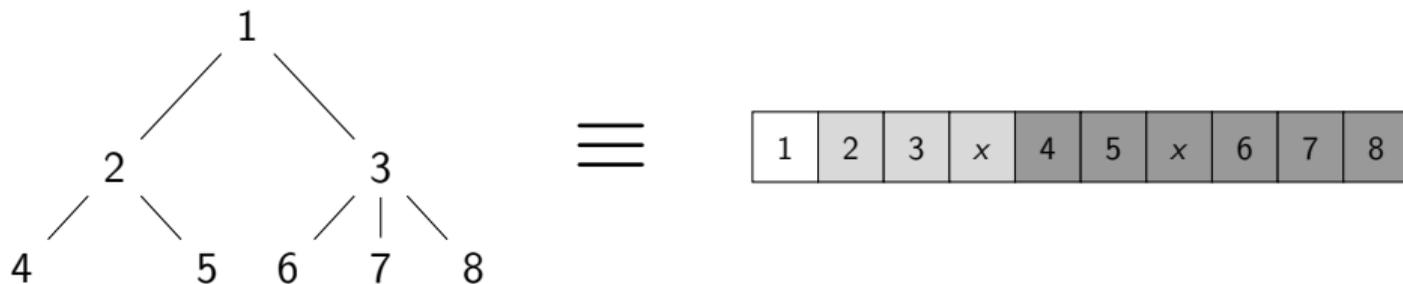


Memory Layout



```
type tree = Empty  
          | Node (tree,int,tree)
```

Memory Layout

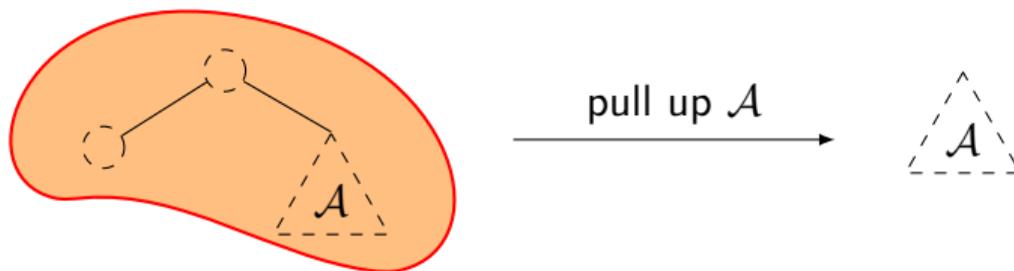


```

type tree = Empty
  | Node2 (int,tree,tree)
  | Node3 (int,tree,tree,tree)

```

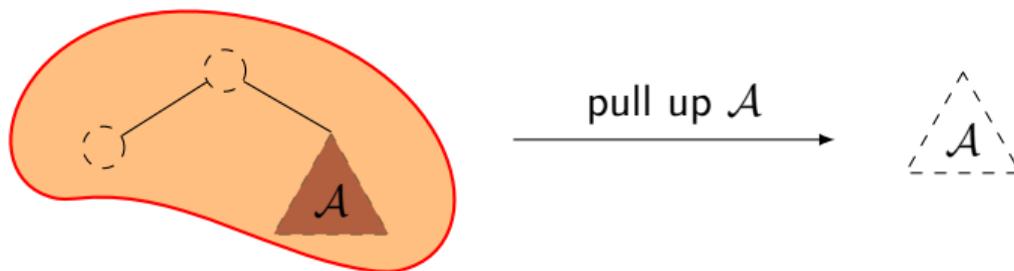
A Language to Describe Structural Transformations



```
type tree = Empty | Node (tree,int,tree)
```

```
pull_up (t : tree) : tree = rewrite t {
  | Node(a,i,Node(b,j,c)) -> Node(b,j,c)
  | Node(a,i,Empty) -> Empty
  | Empty -> Empty
}
```

A Language to Describe Structural Transformations



```
type tree = Empty | Node (tree,int,tree)
```

```
pull_up (t : tree) : tree = rewrite t {
  | Node(a,i,Node(b,j,c)) -> Node(b,j,c)
  | Node(a,i,Empty) -> Empty
  | Empty -> Empty
}
```

A Language to Describe Structural Transformations

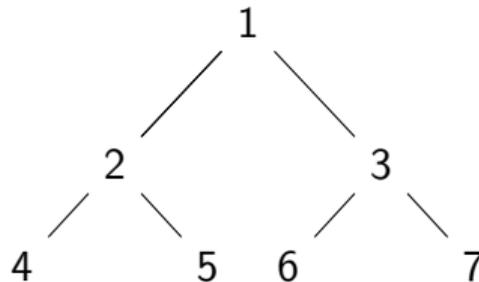


```
type tree = Empty | Node (tree,int,tree)
```

```
pull_up (t : tree) : tree = rewrite t {
  | Node(a,i,Node(b,j,c)) -> Node(b,j,c)
  | Node(a,i,Empty) -> Empty
  | Empty -> Empty
}
```

Our Notations for Locations

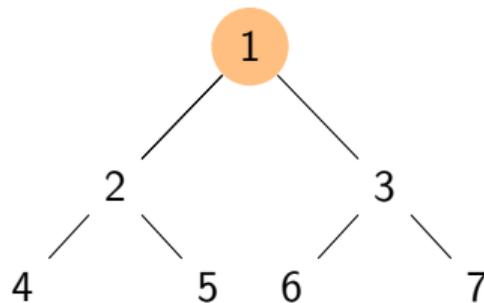
```
type tree = Empty | Node (tree,int,tree)
```



€

Our Notations for Locations

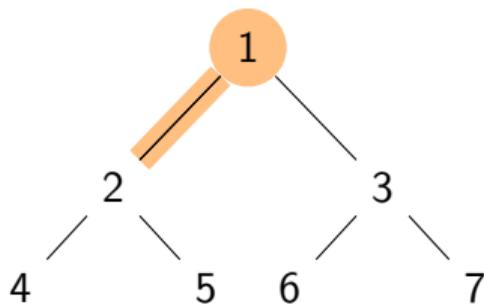
```
type tree = Empty | Node (tree,int,tree)
```



.1/int

Our Notations for Locations

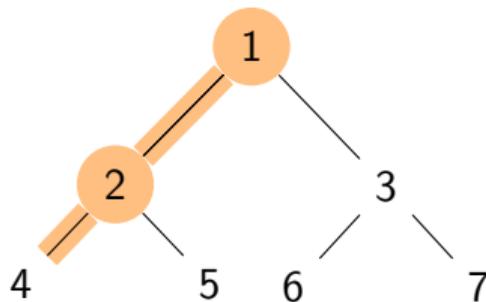
```
type tree = Empty | Node (tree,int,tree)
```



.0/tree

Our Notations for Locations

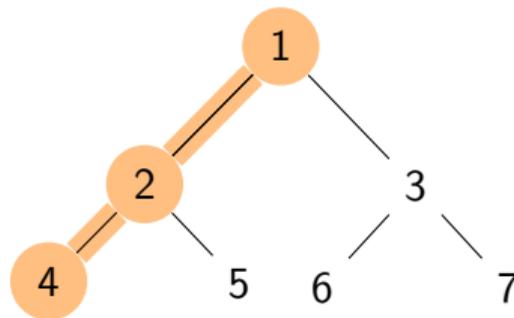
```
type tree = Empty | Node (tree,int,tree)
```



$$.0/tree.0/tree \equiv (.0/tree)^2$$

Our Notations for Locations

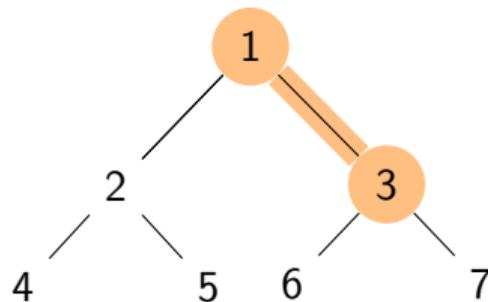
```
type tree = Empty | Node (tree,int,tree)
```



$$(.0/tree)^2.1/int$$

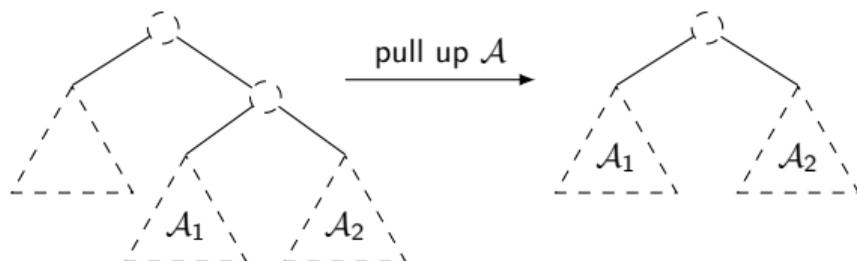
Our Notations for Locations

```
type tree = Empty | Node (tree,int,tree)
```



.2/tree.1/int

Step 1: Compute Subtree Movements

$$\text{Node}(a, i, \text{Node}(b, j, c)) \rightarrow \text{Node}(b, j, c)$$


$$\langle a : tree \mid .0/tree \rightarrow \emptyset \rangle$$

$$\langle i : int \mid .1/int \rightarrow \emptyset \rangle$$

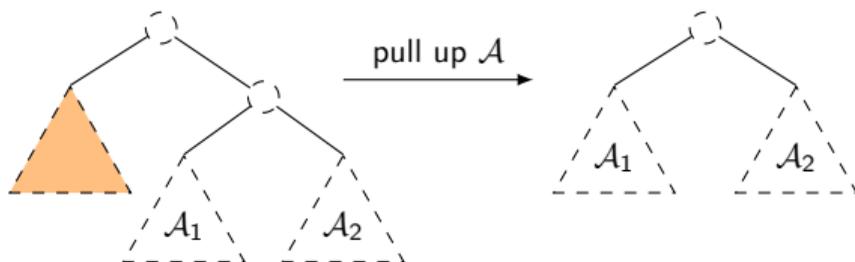
$$\langle b : tree \mid .2/tree.0/tree \rightarrow .0/tree \rangle$$

$$\langle j : int \mid .2/tree.1/int \rightarrow .1/int \rangle$$

$$\langle c : tree \mid .2/tree.2/tree \rightarrow .2/tree \rangle$$

Step 1: Compute Subtree Movements

Node(**a**, i ,Node(b , j , c)) → Node(b , j , c)



$\langle a : tree \mid .0/tree \rightarrow \emptyset \rangle$

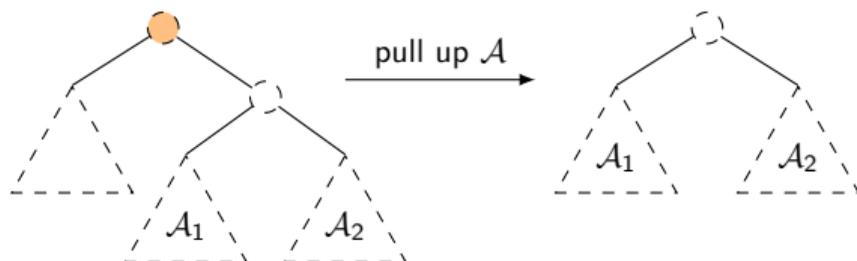
$\langle i : int \mid .1/int \rightarrow \emptyset \rangle$

$\langle b : tree \mid .2/tree.0/tree \rightarrow .0/tree \rangle$

$\langle j : int \mid .2/tree.1/int \rightarrow .1/int \rangle$

$\langle c : tree \mid .2/tree.2/tree \rightarrow .2/tree \rangle$

Step 1: Compute Subtree Movements

$$\text{Node}(a, \mathbf{i}, \text{Node}(b, j, c)) \rightarrow \text{Node}(b, j, c)$$


$$\langle a : tree \mid .0/tree \rightarrow \emptyset \rangle$$

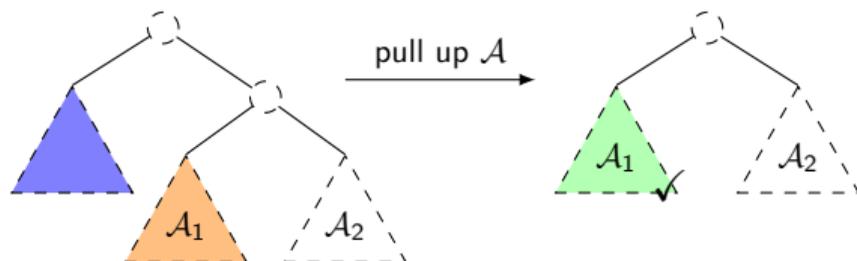
$$\langle i : int \mid \mathbf{.1/int} \rightarrow \emptyset \rangle$$

$$\langle b : tree \mid .2/tree.0/tree \rightarrow .0/tree \rangle$$

$$\langle j : int \mid .2/tree.1/int \rightarrow .1/int \rangle$$

$$\langle c : tree \mid .2/tree.2/tree \rightarrow .2/tree \rangle$$

Step 1: Compute Subtree Movements

$$\text{Node}(a, i, \text{Node}(b, j, c)) \rightarrow \text{Node}(b, j, c)$$


$$\langle a : tree \mid .0/tree \rightarrow \emptyset \rangle$$

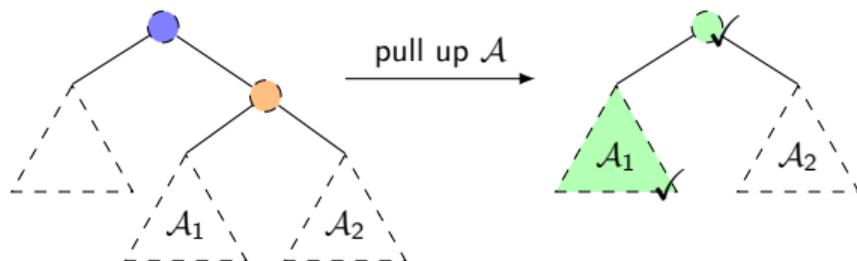
$$\langle i : int \mid .1/int \rightarrow \emptyset \rangle$$

$$\langle b : tree \mid .2/tree.0/tree \rightarrow .0/tree \rangle$$

$$\langle j : int \mid .2/tree.1/int \rightarrow .1/int \rangle$$

$$\langle c : tree \mid .2/tree.2/tree \rightarrow .2/tree \rangle$$

Step 1: Compute Subtree Movements

$$\text{Node}(a, i, \text{Node}(b, j, c)) \rightarrow \text{Node}(b, j, c)$$


$$\langle a : tree \mid .0/tree \rightarrow \emptyset \rangle$$

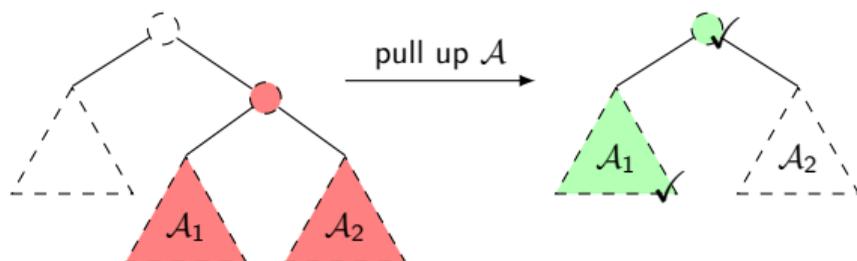
$$\langle i : int \mid .1/int \rightarrow \emptyset \rangle$$

$$\langle b : tree \mid .2/tree.0/tree \rightarrow .0/tree \rangle$$

$$\langle j : int \mid .2/tree.1/int \rightarrow .1/int \rangle$$

$$\langle c : tree \mid .2/tree.2/tree \rightarrow .2/tree \rangle$$

Step 1: Compute Subtree Movements

$$\text{Node}(a, i, \text{Node}(b, j, c)) \rightarrow \text{Node}(b, j, c)$$


$$\langle a : tree \mid .0/tree \rightarrow \emptyset \rangle$$

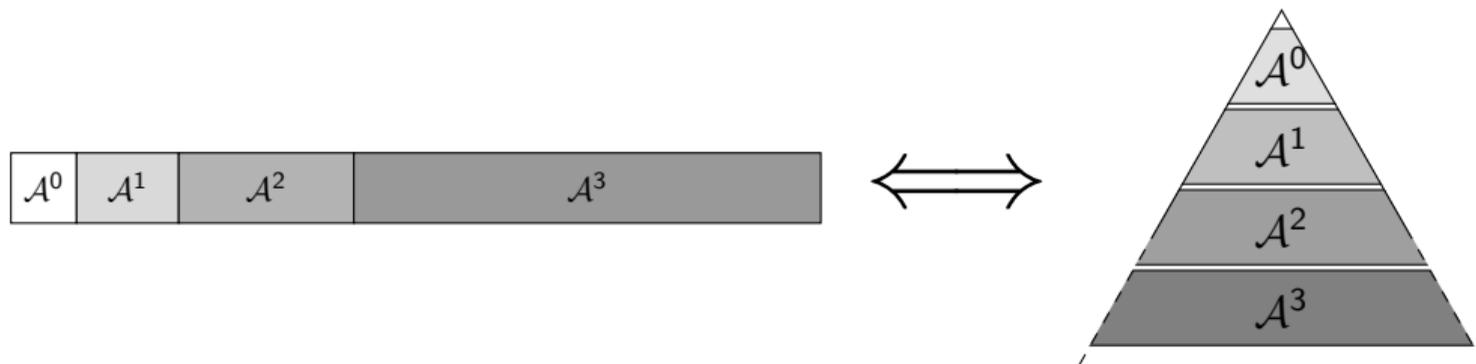
$$\langle i : int \mid .1/int \rightarrow \emptyset \rangle$$

$$\langle b : tree \mid .2/tree.0/tree \rightarrow .0/tree \rangle$$

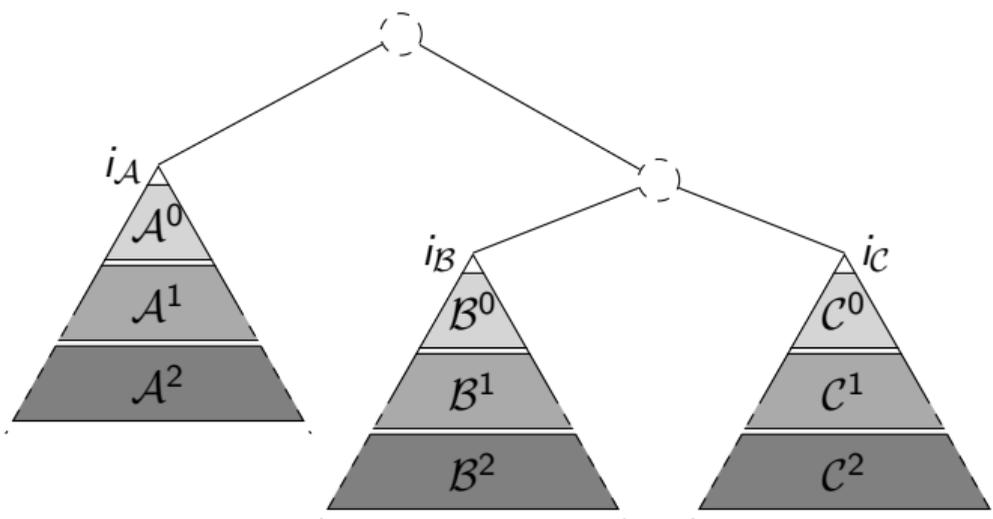
$$\langle j : int \mid .2/tree.1/int \rightarrow .1/int \rangle$$

$$\langle c : tree \mid .2/tree.2/tree \rightarrow .2/tree \rangle$$

Use Layers to Subdivide Memory Movements



Step 2: Layer-aware movements (1/2)

$$\text{Node}(a, i, \text{Node}(b, j, c)) \rightarrow \text{Node}(b, j, c)$$


$$\langle .0/tree. \varphi^{k_0} \rightarrow \emptyset \rangle \quad (a)$$

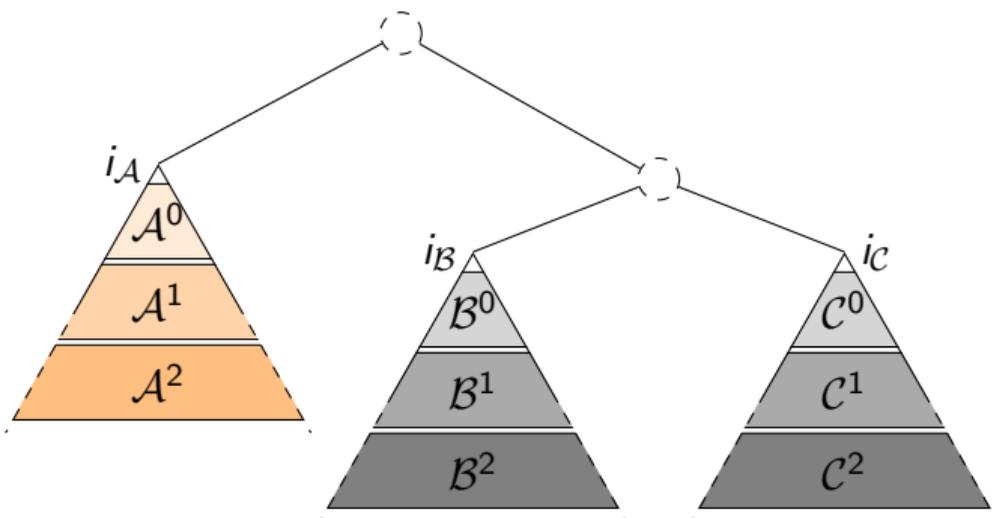
$$\langle .1/int \rightarrow \emptyset \rangle \quad (i)$$

$$\langle .2/tree.0/tree. \varphi^{k_1} \rightarrow .0/tree. \varphi^{k_1} \rangle \quad (b)$$

$$\langle .2/tree.1/int \rightarrow .1/int \rangle \quad (j)$$

Step 2: Layer-aware movements (1/2)

Node(**a**, i ,Node(b , j ,c)) -> Node(b , j ,c)



$(.0/tree.\varphi^{k_0} \rightarrow \emptyset)$

(a)

$(.1/int \rightarrow \emptyset)$

(i)

$(.2/tree.0/tree.\varphi^{k_1} \rightarrow .0/tree.\varphi^{k_1})$

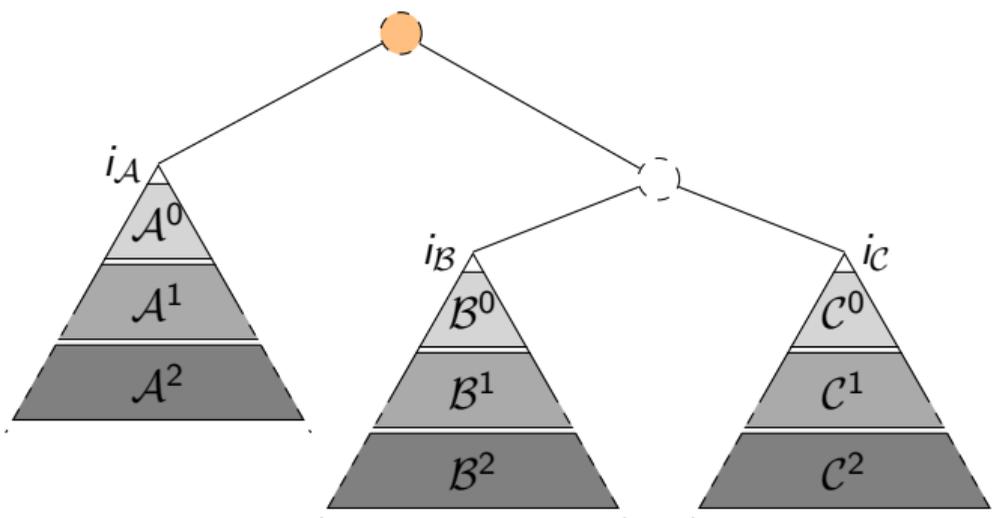
(b)

$(.2/tree.1/int \rightarrow .1/int)$

(j)

Step 2: Layer-aware movements (1/2)

Node(a , **i** , Node(b , j , c)) \rightarrow Node(b , j , c)



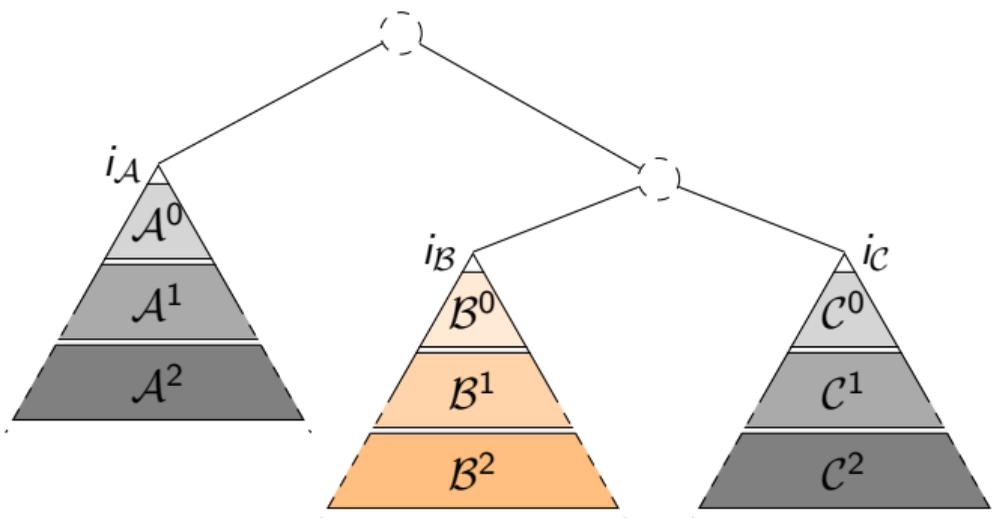
$$\langle \cdot 0/tree. \varphi^{k_0} \rightarrow \emptyset \rangle \quad (a)$$

$$\langle \cdot 1/int \rightarrow \emptyset \rangle \quad (i)$$

$$\langle \cdot 2/tree. 0/tree. \varphi^{k_1} \rightarrow \cdot 0/tree. \varphi^{k_1} \rangle \quad (b)$$

$$\langle \cdot 2/tree. 1/int \rightarrow \cdot 1/int \rangle \quad (j)$$

Step 2: Layer-aware movements (1/2)

$$\text{Node}(a, i, \text{Node}(b, j, c)) \rightarrow \text{Node}(b, j, c)$$


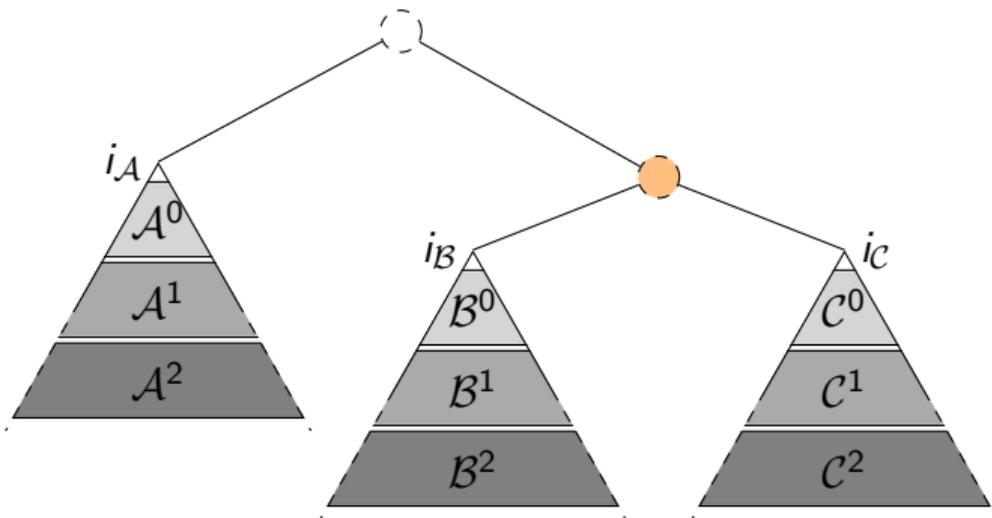
$$\langle .0/tree.\varphi^{k_0} \rightarrow \emptyset \rangle \quad (a)$$

$$\langle .1/int \rightarrow \emptyset \rangle \quad (i)$$

$$\langle .2/tree.0/tree.\varphi^{k_1} \rightarrow .0/tree.\varphi^{k_1} \rangle \quad (b)$$

$$\langle .2/tree.1/int \rightarrow .1/int \rangle \quad (j)$$

Step 2: Layer-aware movements (1/2)

$$\text{Node}(a, i, \text{Node}(b, j, c)) \rightarrow \text{Node}(b, j, c)$$


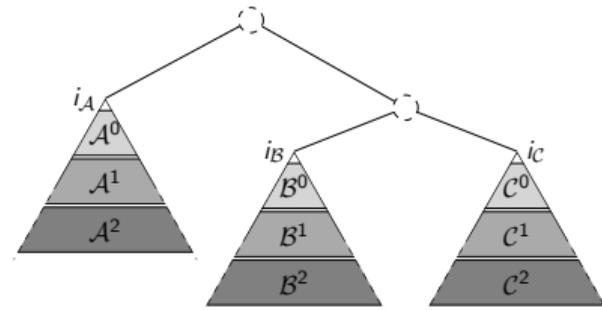
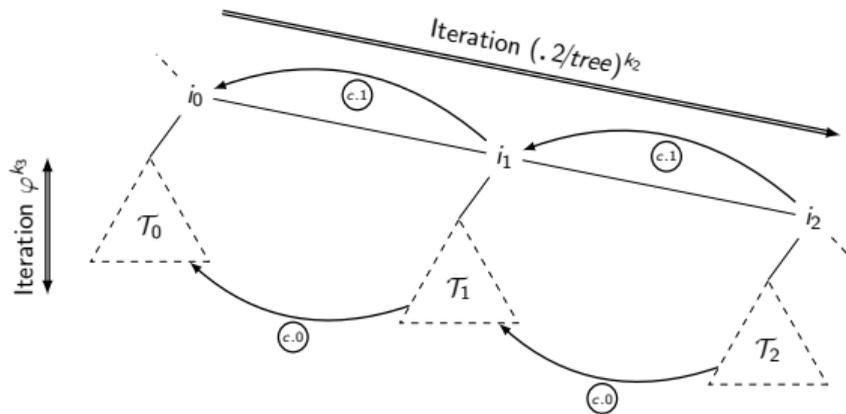
$$\langle .0/tree. \varphi^{k_0} \rightarrow \emptyset \rangle \quad (a)$$

$$\langle .1/int \rightarrow \emptyset \rangle \quad (i)$$

$$\langle .2/tree.0/tree. \varphi^{k_1} \rightarrow .0/tree. \varphi^{k_1} \rangle \quad (b)$$

$$\langle .2/tree.1/int \rightarrow .1/int \rangle \quad (j)$$

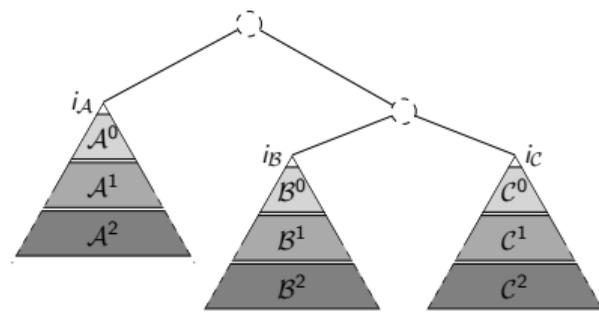
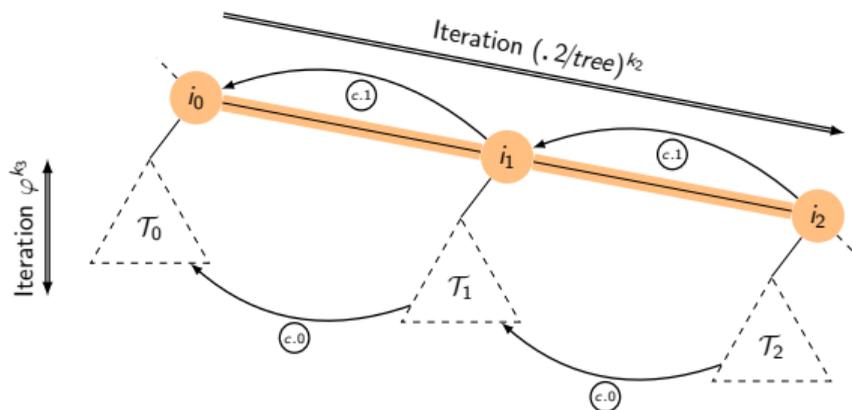
Step 2.5: Layer-aware movements (2/2)



$$\langle (.2/tree)^{k_2+2}.1/int \rightarrow (.2/tree)^{k_2+1}.1/int \rangle \quad (c1)$$

$$\langle (.2/tree)^{k_2+2}.0/tree.\varphi^{k_3} \rightarrow (.2/tree)^{k_2+1}.0/tree.\varphi^{k_3} \rangle \quad (c0)$$

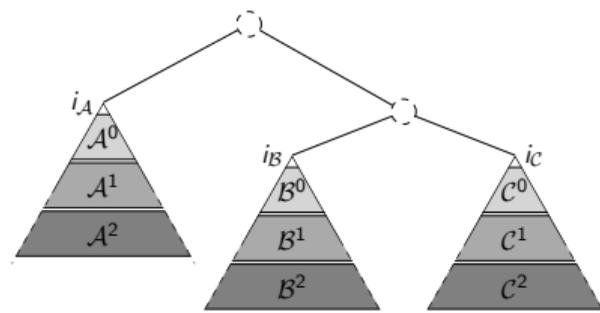
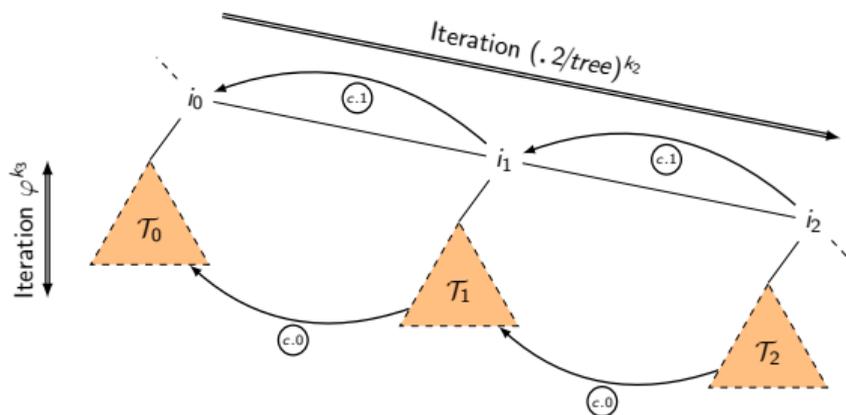
Step 2.5: Layer-aware movements (2/2)



$$\langle \langle (.2/tree)^{k_2+2}.1/int \rightarrow (.2/tree)^{k_2+1}.1/int \rangle \rangle \quad (c1)$$

$$\langle \langle (.2/tree)^{k_2+2}.0/tree.\varphi^{k_3} \rightarrow (.2/tree)^{k_2+1}.0/tree.\varphi^{k_3} \rangle \rangle \quad (c0)$$

Step 2.5: Layer-aware movements (2/2)



$$\langle (.2/tree)^{k_2+2}.1/int \rightarrow (.2/tree)^{k_2+1}.1/int \rangle \quad (c1)$$

$$\langle (.2/tree)^{k_2+2}.0/tree.\varphi^{k_3} \rightarrow (.2/tree)^{k_2+1}.0/tree.\varphi^{k_3} \rangle \quad (c0)$$

Contents

- 1 Presentation of REW
 - Structural Transformations
 - Coarse Memory Movements
 - Fine-Grain Memory Movements
- 2 Scheduling Memory Movements
 - Memory Movements Domains
 - Inter-Movement Dependencies
 - Schedule Computation
- 3 Code Generation
 - Application of the Algorithm to the Running Example
- 4 Some Manual Experiments
 - AVL Trees
 - Decomposition of Rotations
 - Benchmarks

Some definitions

Definition (Paths and memory movements)

k	$\in \text{ItVars}$	(Iteration variables)
π	$::= \ell . \pi \mid \ell^k . \pi \mid \varphi^k \mid \varepsilon$	(Path)
m_π	$::= (\pi \rightarrow \pi')$	(Move)

Definition (Admissible length of a path)

Given a path π , its *admissible length*, written $|\pi|$, is:

$$\begin{array}{ll}
 |\ell . \pi| = |\ell| + |\pi| & |\varphi^k| = k \\
 |\ell^k . \pi| = |\ell| * k + |\pi| & |\varepsilon| = 0
 \end{array}$$

Domain of a move

Definition (Domain of a move)

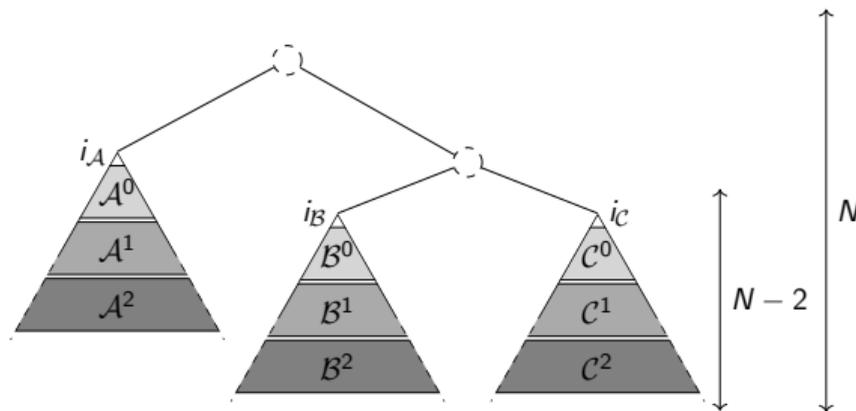
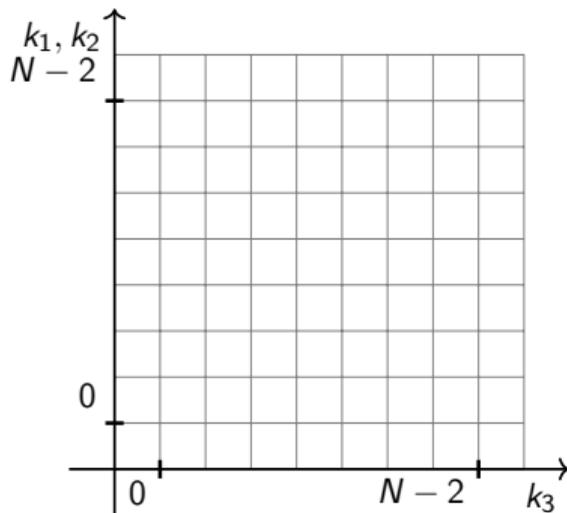
We consider a move $m = (\pi \rightarrow \pi')$. The domain of m is written \mathcal{D}_m and defined

$$\mathcal{D}_m = \left\{ \vec{k} \mid (0 \leq |\pi|(\vec{k}) \leq N) \wedge (0 \leq |\pi'|(\vec{k}) \leq N) \wedge (\vec{0} \leq \vec{k}) \right\}$$

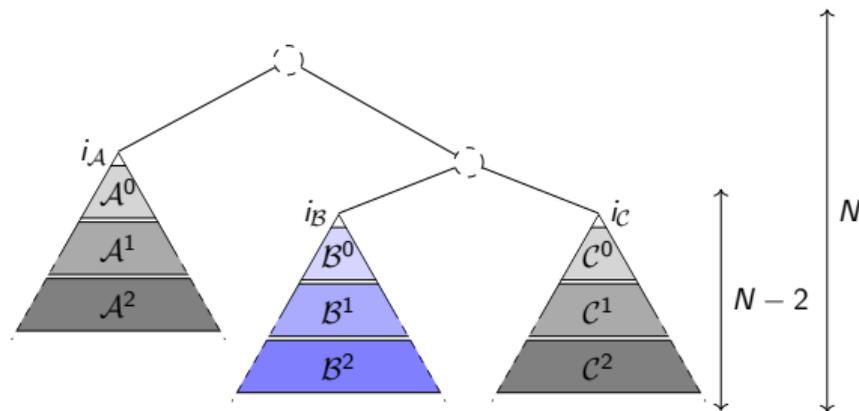
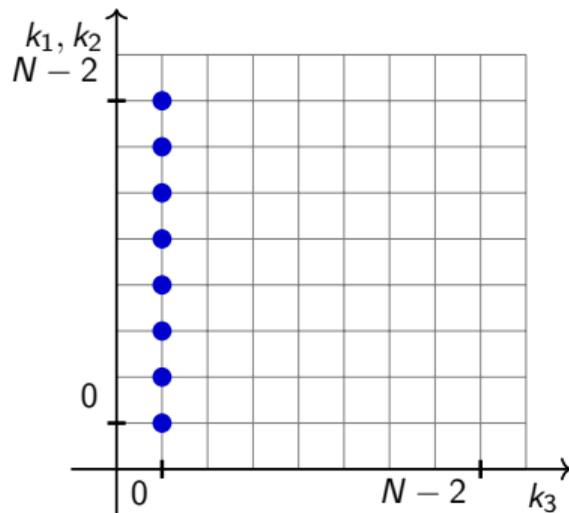
We write D_m and \vec{d}_m such that $\mathcal{D}_m = \left\{ \vec{k} \mid D_m \vec{k} + \vec{d}_m \geq \vec{0} \right\}$

Since $|\pi|$ is a linear form on \vec{k} , the domain can also be defined as a polyhedron.

Step 3: Characterizing Memory Movements



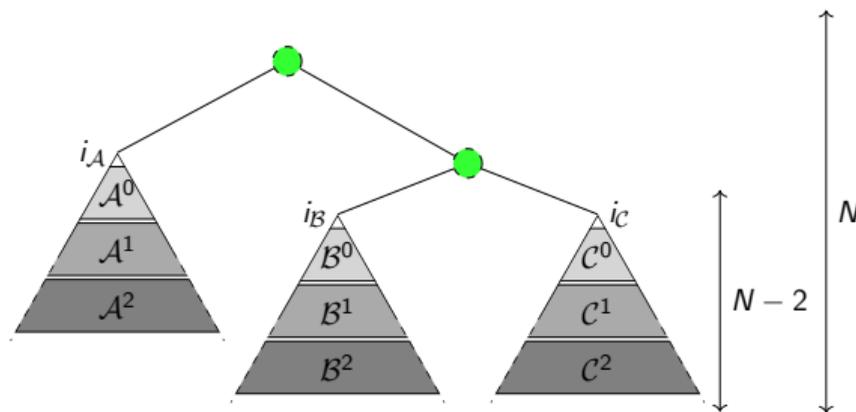
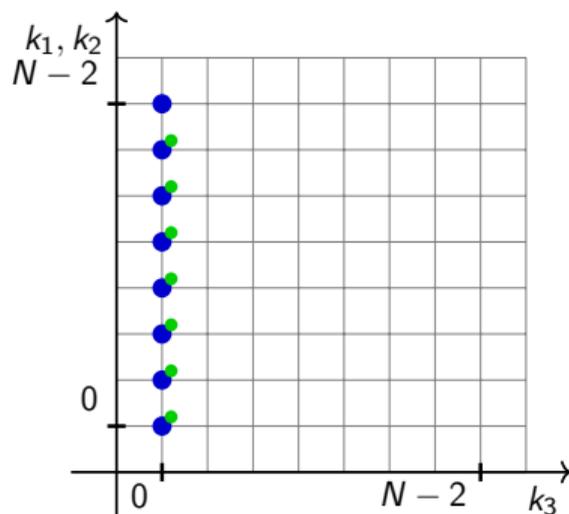
Step 3: Characterizing Memory Movements



$$(.2/tree.0/tree.\varphi^{k_1} \rightarrow .0/tree.\varphi^{k_1})$$

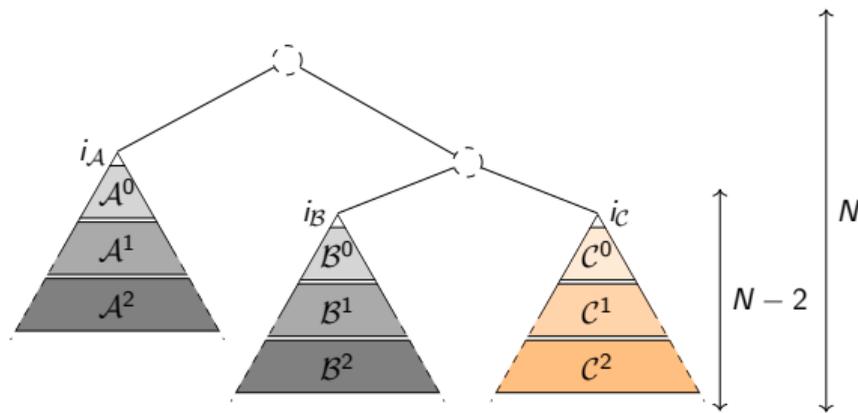
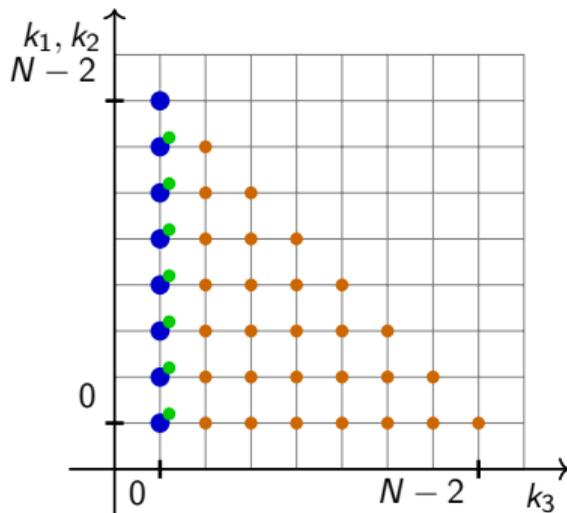
(b)

Step 3: Characterizing Memory Movements



$$\langle (.2/tree)^{k_2+2} . 1/int \rightarrow (.2/tree)^{k_2+1} . 1/int \rangle \quad (c1)$$

Step 3: Characterizing Memory Movements



$$\left((.2/tree)^{k_2+2} .0/tree . \varphi^{k_3} \rightarrow (.2/tree)^{k_2+1} .0/tree . \varphi^{k_3} \right) \quad (c0)$$

Dependencies between moves

Definition ((R/W) Dependencies between moves)

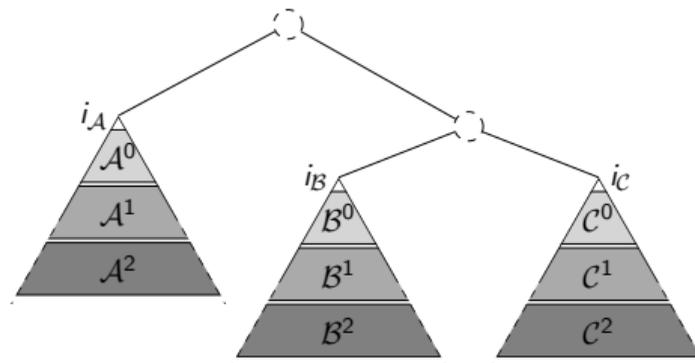
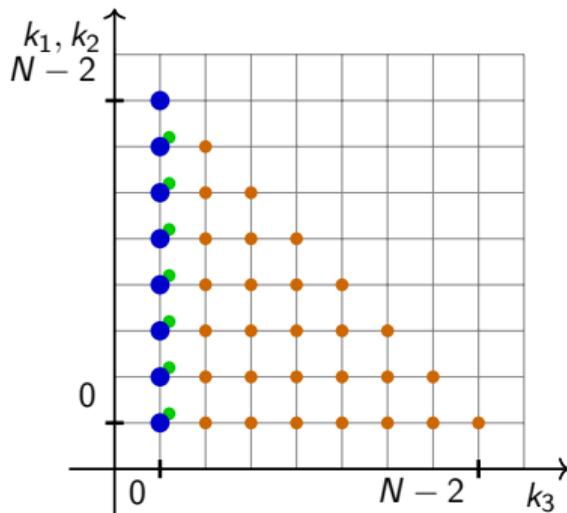
Let $m = (\pi_p \rightarrow \pi_e)$ and $m' = (\pi'_p \rightarrow \pi'_e)$ be two moves. Given $\mathcal{L}(\pi)$ the set of locations of π , we have:

$$\mathcal{Q}_{(m,m')} = \left\{ \begin{pmatrix} \vec{k} \\ \vec{k}' \end{pmatrix} \mid \exists \ell \in \mathcal{L}(\pi_p(\vec{k})) \cap \mathcal{L}(\pi'_e(\vec{k}')) \right\}$$

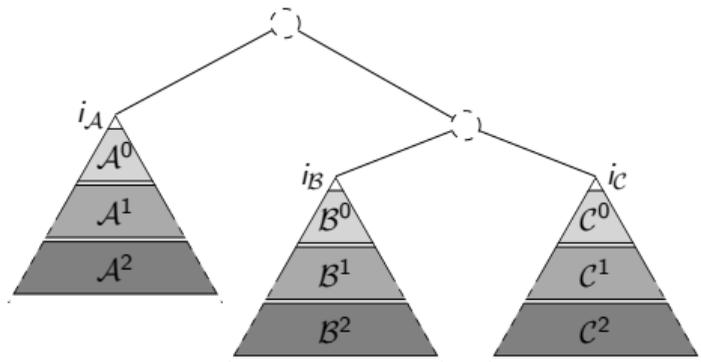
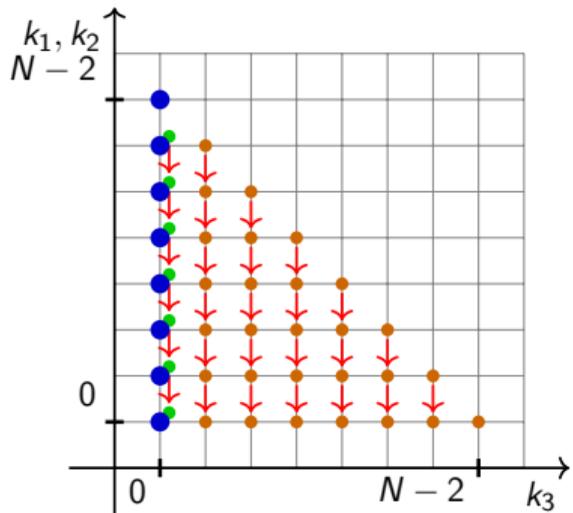
Lemma

$\mathcal{Q}_{(m,m')}$ is a union of polyhedrons and computing its finite representation is decidable.

Characterizing Memory Movements (Dependencies)



Characterizing Memory Movements (Dependencies)



Schedule Definition

Definition (Schedule & Constraints)

A *schedule* for the graph $(\mathcal{M}, \mathcal{T})$ is a function $\rho : \mathcal{M} \times \mathbb{Z}^n \rightarrow \mathbb{N}^d$, from the graph vertices to \mathbb{N}^d , which is positive:

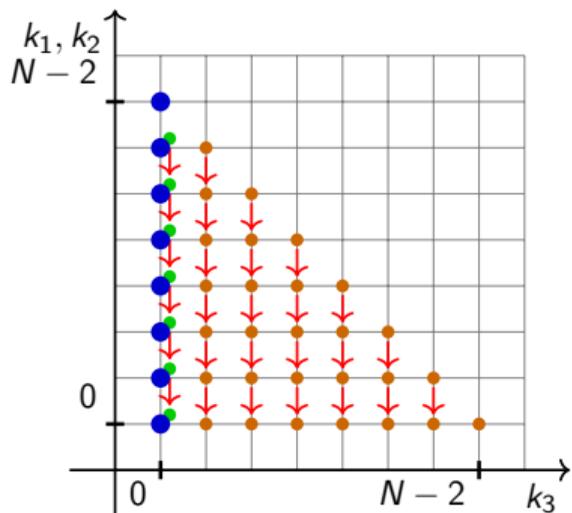
$$\vec{k} \in \mathcal{D}_m \Rightarrow \rho(m, \vec{k}) \geq \vec{0} \text{ (component-wise)} \quad (\text{Positivity})$$

and whose values *strictly* increase (according to \preceq_d , the standard lexicographic order on integer vectors) at each edge $t = (m, m') \in \mathcal{T}$:

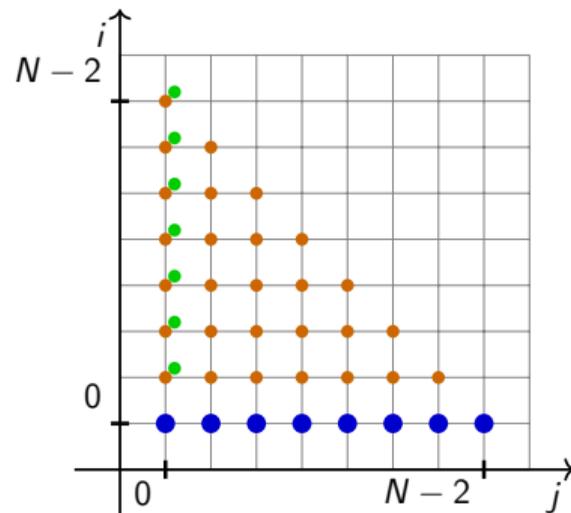
$$\mathcal{Q}_{m,m'}(\vec{k}, \vec{k}') \neq \emptyset \Rightarrow \rho(m, \vec{k}) \prec_d \rho(m', \vec{k}') \quad (\text{Increasing})$$

It is said *affine* if it is affine in the second parameter (the variables \vec{k}). If $d > 0$ the schedule is said to be multi-dimensional of dimension d .

Step 4: Scheduling as an Integer Linear Program Problem



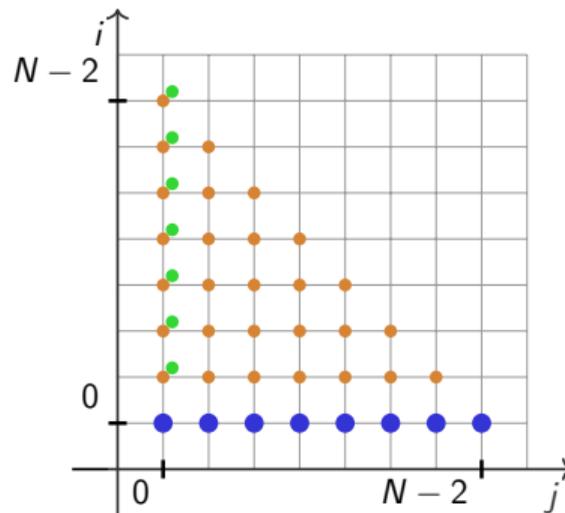
Scheduling by Farkas
 →
 and ILP Solver



Contents

- 1 Presentation of REW
 - Structural Transformations
 - Coarse Memory Movements
 - Fine-Grain Memory Movements
- 2 Scheduling Memory Movements
 - Memory Movements Domains
 - Inter-Movement Dependencies
- 3 Code Generation
 - Schedule Computation
 - Application of the Algorithm to the Running Example
- 4 Some Manual Experiments
 - AVL Trees
 - Decomposition of Rotations
 - Benchmarks

Step 5: From Schedule to Code

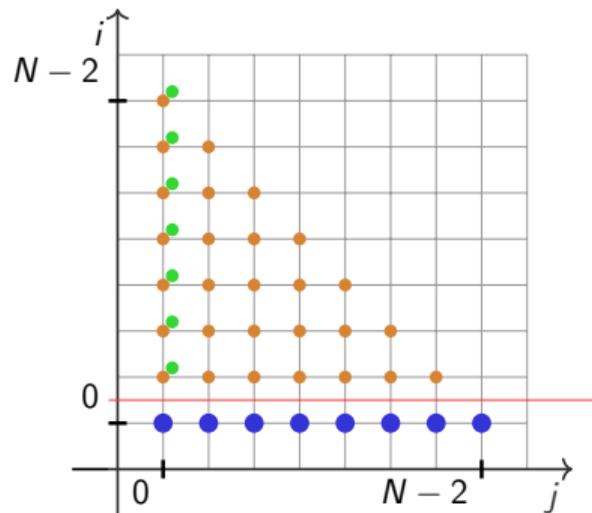


Application of Quilleré's Algorithm

Step 5: From Schedule to Code

```
for (i = 0 ; i <= 0 ; i += 1)
```

```
for (i = 1 ; i <= N-2 ; i += 1)
```

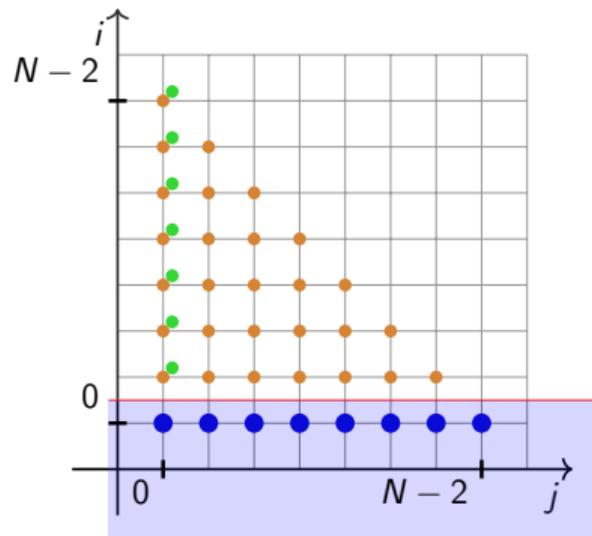


Application of Quilleré's Algorithm

Step 5: From Schedule to Code

```
for (i = 0 ; i <= 0 ; i += 1)
  for (j = 0 ; j <= N-2 ; j += 1)

for (i = 1 ; i <= N-2 ; i += 1)
```



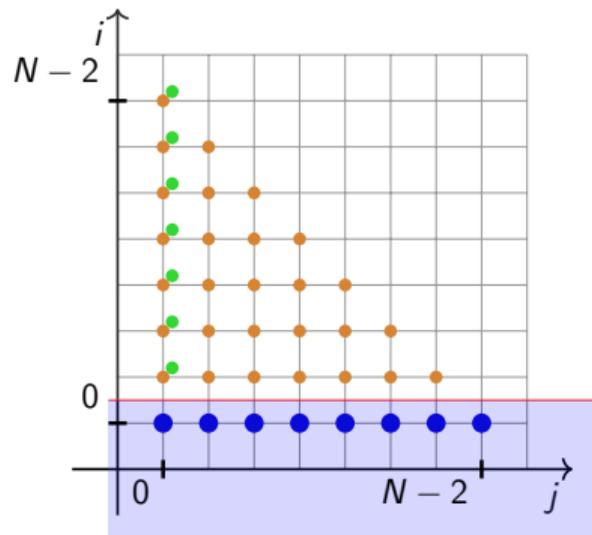
Application of Quilleré's Algorithm

Step 5: From Schedule to Code

```

for (i = 0 ; i <= 0 ; i += 1)
  for (j = 0 ; j <= N-2 ; j += 1)
    (.2/tree.0/tree.φj → .0/tree.φj) // b
for (i = 1 ; i <= N-2 ; i += 1)

```

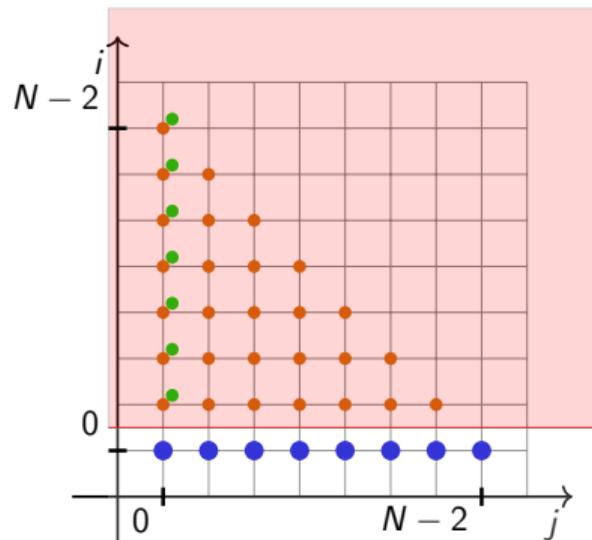


Application of Quilleré's Algorithm

Step 5: From Schedule to Code

```
for (i = 0 ; i <= 0 ; i += 1)
  for (j = 0 ; j <= N-2 ; j += 1)
    (.2/tree.0/tree.φj → .0/tree.φj) // b
for (i = 1 ; i <= N-2 ; i += 1)
  for (j = 0 ; j <= 0 ; j += 1)

  for (j = 0 ; j <= N - i - 2 ; j += 1)
```



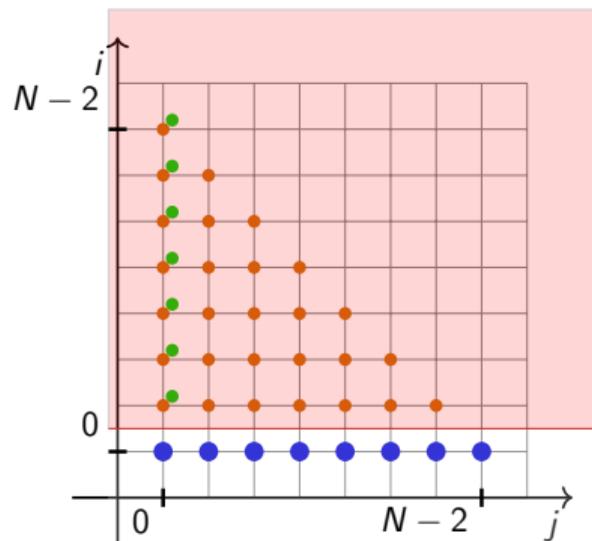
Application of Quilleré's Algorithm

Step 5: From Schedule to Code

```

for (i = 0 ; i <= 0 ; i += 1)
  for (j = 0 ; j <= N-2 ; j += 1)
     $(.2/tree.0/tree.\varphi^j \rightarrow .0/tree.\varphi^j)$  // b
for (i = 1 ; i <= N-2 ; i+= 1)
  for (j = 0 ; j <= 0 ; j += 1)
     $( (.2/tree)^{i+1}.1/int \rightarrow (.2/tree)^i.1/int )$  //c1
  for (j = 0 ; j <= N - i - 2 ; j += 1)
     $( (.2/tree)^{i+1}.0/tree.\varphi^j \rightarrow (.2/tree)^i.0/tree.\varphi^j )$  //c0

```



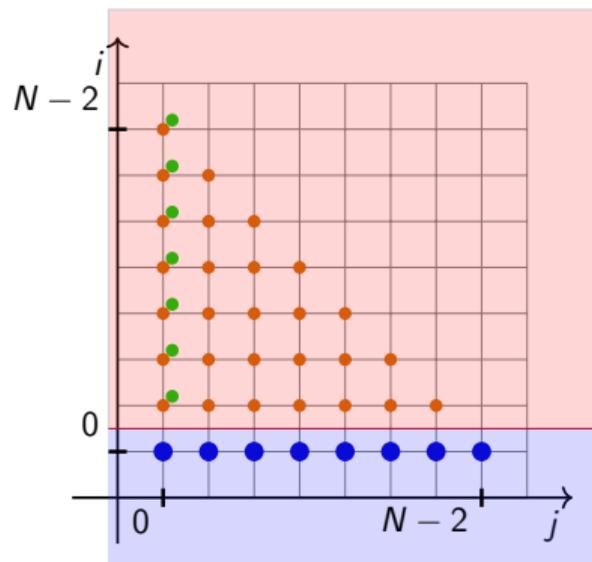
Application of Quilleré's Algorithm

Step 5: From Schedule to Code

```

for (j = 0 ; j <= N-2 ; j += 1)
   $\langle (.2/tree . 0/tree . \varphi^j \rightarrow . 0/tree . \varphi^j) \rangle$  // b
for (i = 1 ; i <= N-2 ; i += 1)
   $\langle (.2/tree)^{i+1} . 1/int \rightarrow (.2/tree)^i . 1/int \rangle$  // c1
  for (j = 0 ; j <= N - i - 2 ; j += 1)
     $\langle (.2/tree)^{i+1} . 0/tree . \varphi^j \rightarrow (.2/tree)^i . 0/tree . \varphi^j \rangle$  // c0

```



Application of Quilleré's Algorithm

Conclusion & Future Work

- A promising technique to compile pattern matching to in-place transformation:
 - Based on term-rewriting
 - Use linear equations on regexps!
 - Schedule the operations (pipelining, parallelization)
 - Generate code

Future Work

- Extend the input language
 - recursion
 - guards
- Improve the code generation
 - parallel code generation

Conclusion & Future Work

- A promising technique to compile pattern matching to in-place transformation:
 - Based on term-rewriting
 - Use linear equations on regexps!
 - Schedule the operations (pipelining, parallelization)
 - Generate code

Future Work

- Extend the input language
 - recursion
 - guards
- Improve the code generation
 - parallel code generation

Contents

- 1 Presentation of REW
 - Structural Transformations
 - Coarse Memory Movements
 - Fine-Grain Memory Movements
- 2 Scheduling Memory Movements
 - Memory Movements Domains
 - Inter-Movement Dependencies
- 3 Code Generation
 - Schedule Computation
 - Application of the Algorithm to the Running Example
- 4 Some Manual Experiments
 - AVL Trees
 - Decomposition of Rotations
 - Benchmarks

AVL Trees

Definition

AVL Trees AVL Trees are self-rebalancing binary search trees relying on a rotation mechanism.

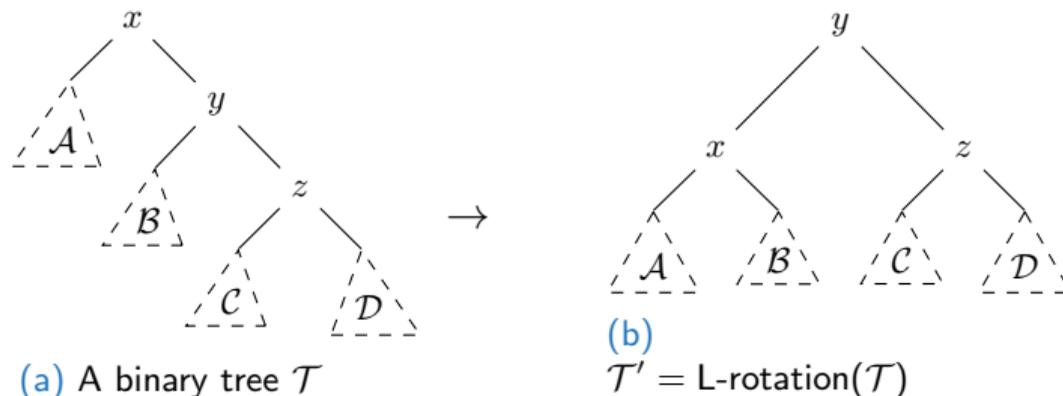


Figure: Left (L) rotation applied on to the unbalanced tree \mathcal{T}

AVL Trees

Definition

AVL Trees AVL Trees are self-rebalancing binary search trees relying on a rotation mechanism.

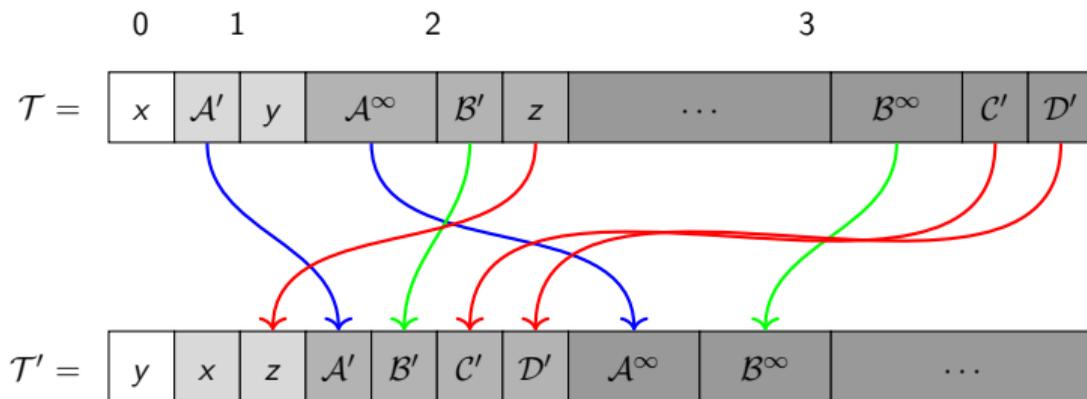


Figure: L-rotation on a tarbre

Low-level operations

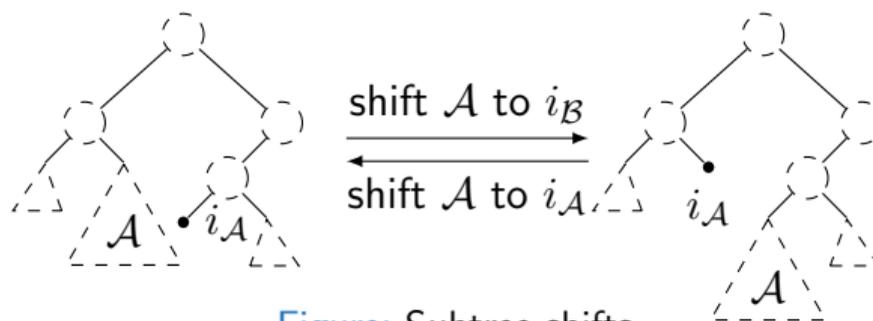


Figure: Subtree shifts

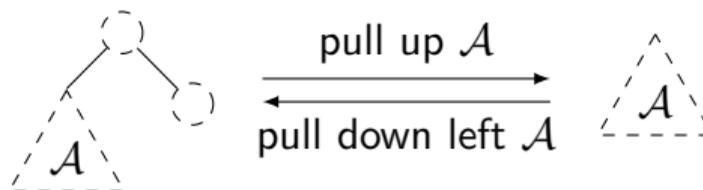
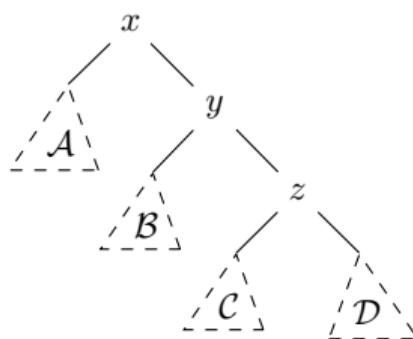
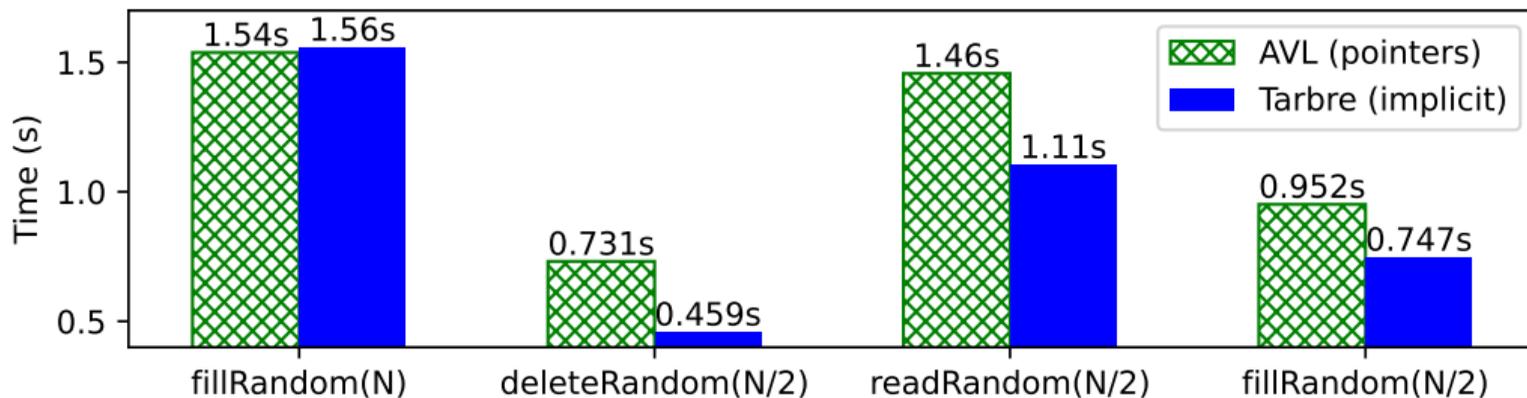


Figure: Subtree pull-ups and pull-downs

Rotations as low-level operations



Rotation	Right	Left
Steps	<ol style="list-style-type: none"> 1. pull-down-right($\mathcal{T}, i_{\mathcal{D}}$) 2. shift($\mathcal{T}, i_{\mathcal{C}}, i_{\mathcal{C}} + 1$) 3. pull-up($\mathcal{T}, i_{\mathcal{Z}}$) 4. move-values($\mathcal{T}, i_{\mathcal{X}}, i_{\mathcal{Y}}, i_{\mathcal{Z}}$) 	<ol style="list-style-type: none"> 1. pull-down-left($\mathcal{T}, i_{\mathcal{A}}$) 2. shift($\mathcal{T}, i_{\mathcal{B}}, i_{\mathcal{B}} - 1$) 3. pull-up($\mathcal{T}, i_{\mathcal{Z}}$) 4. move-values($\mathcal{T}, i_{\mathcal{X}}, i_{\mathcal{Y}}, i_{\mathcal{Z}}$)

Benchmark on a key-value store scenario ($N = 1M \approx 2^{20}$)

OCaml Type Checker

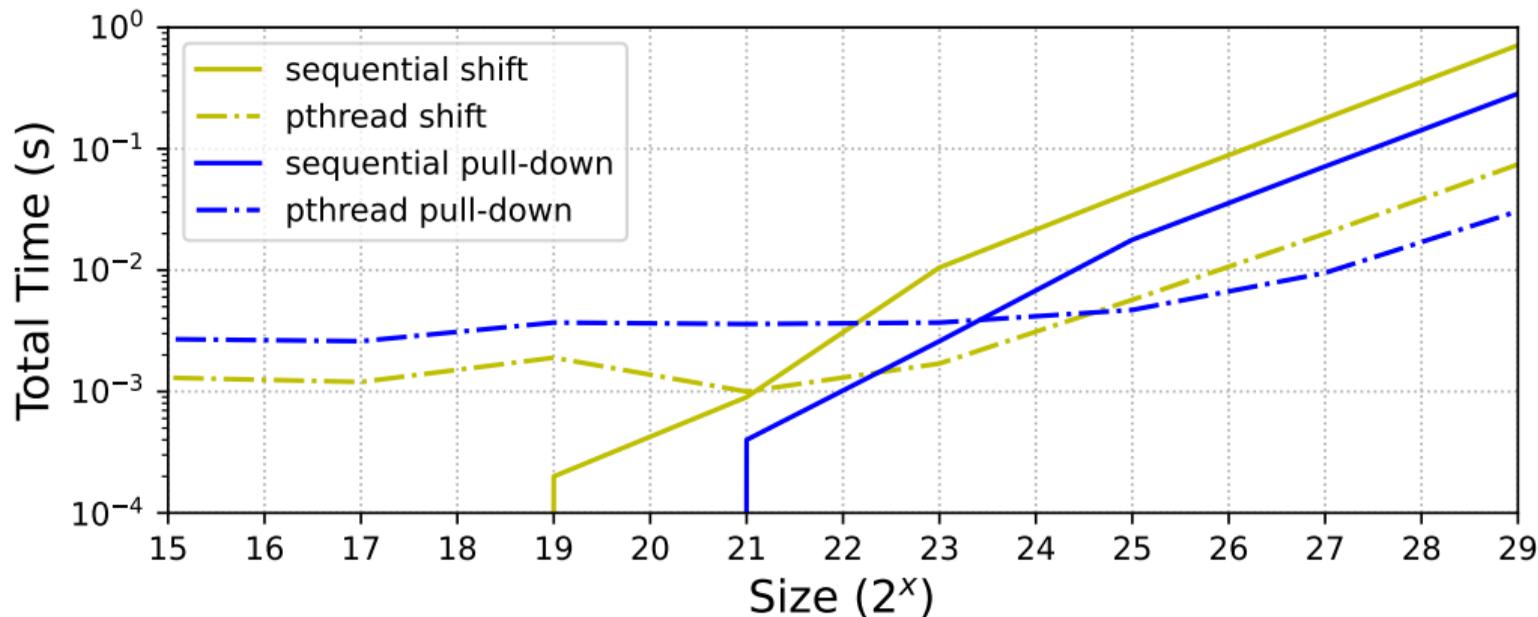
```

1223 <- add1(0,kind_478,layout_479) // Tree creation
1224 <- add(1223,arr_483) // Tree extension
find(1224,arr_483) // Lookup in tree 1224
find(1224,c_init_460)
...
free(1224) // Tree 1224 is freed
find(1223,create_430) // Lookup in tree 1223

```

	AVL (pointers)	tarbre (implicit)	std::set
Time (s)	5.64s	5.04s	5.78s
Memory	12Mo	13Mo	12Mo

Parallel Benchmark



Conclusion

- Summary:
 - A simple decomposition of rotations into simpler operations
 - Performance which are comparable on a sequential workload
 - Micro benchmarks show that there is room for improvement
- Current work:
 - Working on a complete parallel scenario

Contents

- 1 Presentation of REW
 - Structural Transformations
 - Coarse Memory Movements
 - Fine-Grain Memory Movements
- 2 Scheduling Memory Movements
 - Memory Movements Domains
 - Inter-Movement Dependencies
 - Schedule Computation
- 3 Code Generation
 - Application of the Algorithm to the Running Example
- 4 Some Manual Experiments
 - AVL Trees
 - Decomposition of Rotations
 - Benchmarks