Verification of efficient C arithmetic algorithms with Why3

Raphaël Rieu-Helft

joint work with Guillaume Melquiond and Claude Marché

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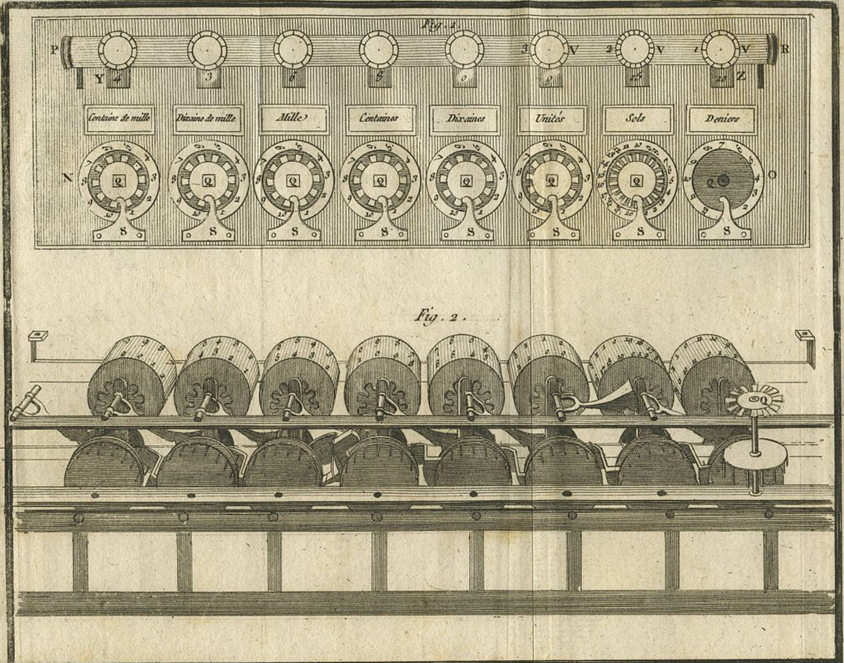

P. B. Bradel del et Sculp.

Computer arithmetic, integer representation

Usual integer representation

- Machine word: string of k bits, k depends on the architecture
- Typically $k = 64$ or $k = 32$
- A machine word can represent any integer between 0 and $2^k - 1$

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What about larger numbers?

- Required for cryptography, computer algebra systems...

Arbitrary-precision arithmetic

Integer representation

large integer $\equiv$ array of unsigned integers $a_0 \ldots a_{n-1}$ called limbs

$$\texttt{value}(a, n) = \sum_{i=0}^{n-1} a_i \beta^i \qquad 0 \leq a_i < \beta \qquad \beta = 2^{64}$$

Arbitrary-precision arithmetic

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The GNU Multiple Precision library (GMP)

- Free software, widely used arbitrary-precision arithmetic library
- State-of-the-art algorithms written in C

Motivation

Decrementing a long integer by 1 (simplified from `mpn_decr_u`)

```
#define mpn_decr_1(x)              \
  mp_ptr __x = (x);                \
  while ((*(__x++))-- == 0) ;
```

- Hard-to-read single-line code from the GMP library
- Can the program crash? (safety)
- Does it compute the right value? (functional correctness)

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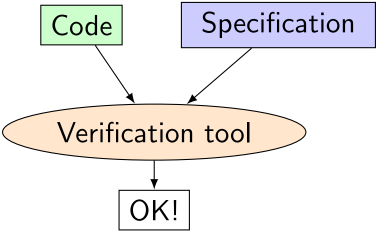
How to verify this?

Functional verification in a nutshell



Informal specification (incomplete)

```
// requires: x valid over some length sz
// requires: value x sz >= 1
// ensures:  value x sz = old (value x sz) - 1
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Next steps:

- formalize the specification
- check whether the program matches the specification

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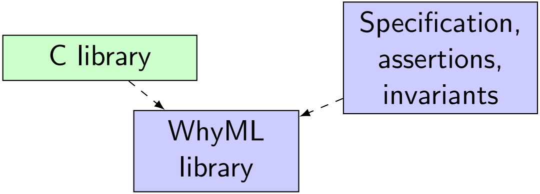
How to do so efficiently?

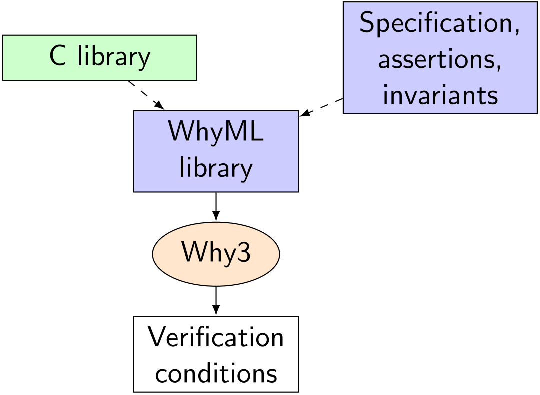
The Why3 workflow

C library

The Why3 workflow



The Why3 workflow



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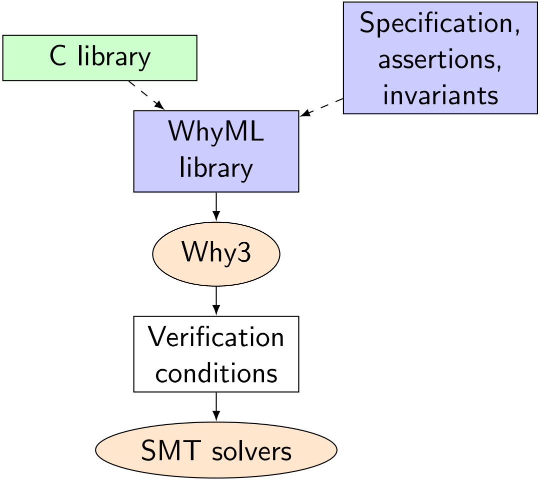

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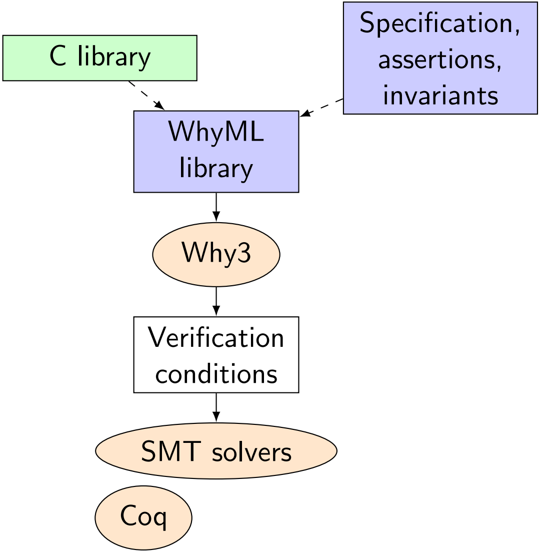

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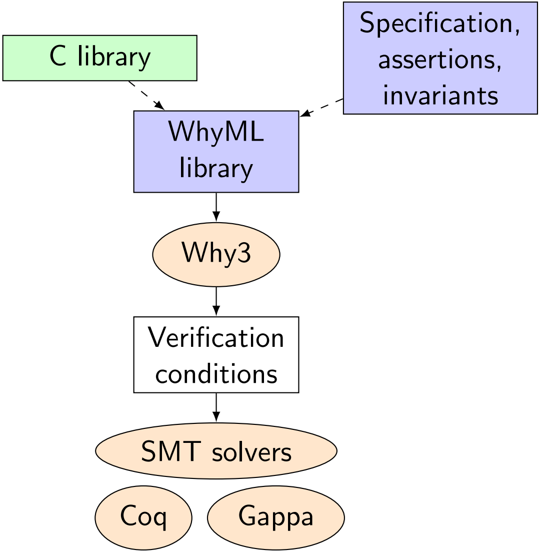

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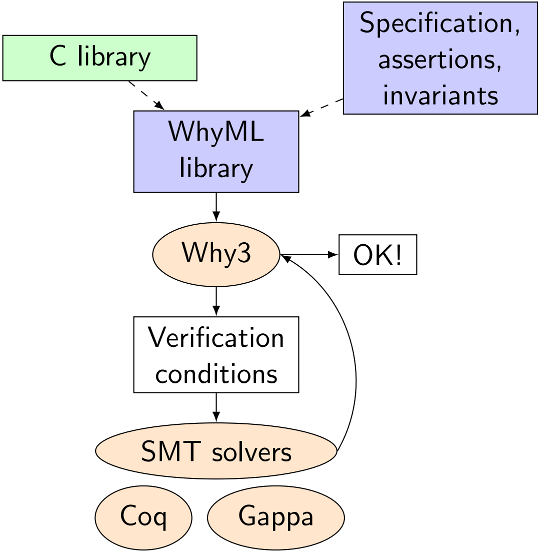

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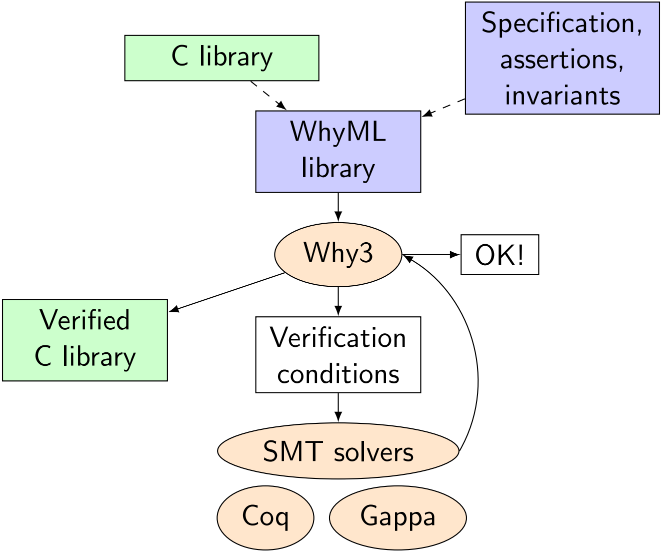


Verifying mpn_decr_1

Original macro (simplified from 18-line mpn_decr_u)

```
#define mpn_decr_1(x)                \
  mp_ptr __x = (x);                  \
  while ((*(__x++))-- == 0) ;
```

Translation to WhyML

```
let wmpn_decr_1 (x: ptr uint64) (ghost sz: int32): unit
  requires { valid x sz }
  requires { 1 <= value x sz }
  ensures  { value x sz = value (old x) sz - 1 }
=
  let ref lx = 0 in
  let ref xp = incr x 0 in
  while lx = 0 do
    lx <- get xp;
    set xp (sub_mod lx 1);
    xp <- incr xp 1;
  done
```

Verifying mpn_decr_1

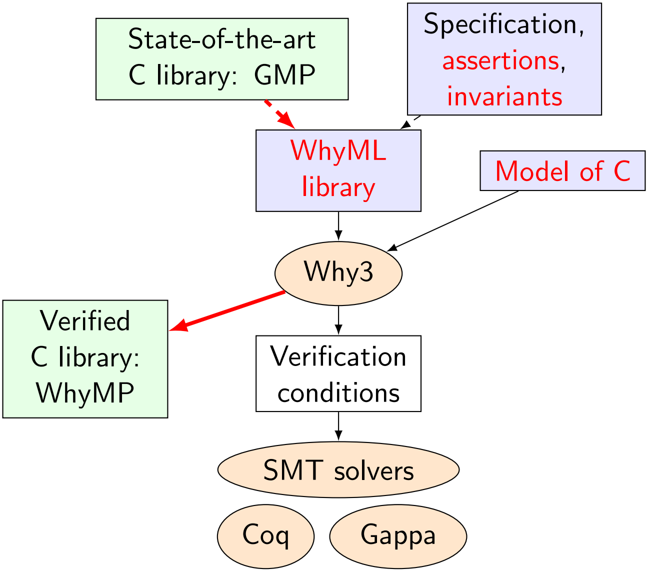
Original macro (simplified from 18-line mpn_decr_u)

```
#define mpn_decr_1(x)                \
  mp_ptr __x = (x);                  \
  while ((*(__x++))-- == 0) ;
```

Extraction to C

```
void wmpn_decr_1(uint64_t *x) {
  uint64_t lx, *xp, res;
  lx = 0;
  xp = x + 0;
  while (lx == 0) {
    lx = *xp;
    res = lx - 1;
    *xp = res;
    xp = xp + 1;
  }
}
```

Overview


State-of-the-art C library: GMP
Specification, assertions, invariants
WhyML library
Model of C
Why3
Verified C library: WhyMP
Verification conditions
SMT solvers
Coq
Gappa

Plan

1. Introduction
2. Memory model and extraction
3. An algorithm: long division
4. WhyMP
5. Conclusion, perspectives

Memory model: goals and challenges

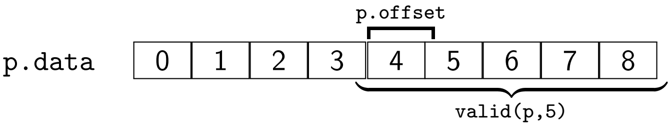
Goals

- Accurate transcription of C programs in WhyML
- Tractable proofs

Challenges

- No native notion of pointers in WhyML
- Alias handling:
  - Aliased pointers
  - Function arguments that may or may not be aliased

Memory model




```
type ptr 'a = abstract { mutable data: array 'a ; offset: int }

predicate valid (p:ptr 'a) (sz:int) =
  0 ≤ sz ∧ 0 ≤ p.offset ∧ p.offset + sz ≤ p.data.length

val malloc (sz:uint32) : ptr 'a          (* malloc(sz * sizeof('a)) *)
  ensures { is_not_null result → valid result sz }
  ...

val free (p:ptr 'a) : unit               (* free(p) *)
  ...
```

Alias control

aliased C pointers	$\Leftrightarrow$	point to the same memory object
aliased WhyML pointers	$\Leftrightarrow$	shared value in the data field

```
type ptr 'a = abstract { mutable data: array 'a ; offset: int }

val incr (p:ptr 'a) (ofs:int32): ptr 'a          (* p + ofs  *)
  alias    { result.data with p.data }
  ensures  { result.offset = p.offset + ofs }
  ...

val free (p:ptr 'a) : unit
  requires { p.offset = 0 }
  writes   { p.data }
  ensures  { p.data.length = 0 }
```

Extraction mechanism

Goals

- Straightforward extraction (trusted)
- Performance: no added complexity, no closures or indirections
- Predictable output
- Tradeoff: handle only a small, C-like fragment of WhyML

✓ loops, references
✓ records
✓ machine integers
✓ manual memory management

✗ polymorphism, abstract types
✗ higher order
✗ mathematical integers
✗ garbage collection

Exceptions and break/return

- Recognize `break`- and `return`-like patterns
- Reject other exceptions

```
exception B

try
  while ... do
    ...
    if (...) then raise B;
    ...
  done
with B → ()
end
```

```
exception R of t

let f (...) : t =
...
try
  ...
  raise (R e)
  ...
with R v → v
end
```

Tuple return values

```
let f (x:int32) : (int32, int32)
  = x + 1, x + 2

let g (x:int32) =
  let (y, z) = f x in
  y + z
```

```
struct __f_result {
  int32_t __field_0;
  int32_t __field_1;
};

struct __f_result f(int32_t x) {
  struct __f_result result;
  result.__field_0 = x + 1;
  result.__field_1 = x + 2;
  return result;
}

int32_t g(int32_t x) {
  int32_t y, z;
  struct __f_result struct_res;
  struct_res = f(x);
  y = struct_res.__field_0;
  z = struct_res.__field_1;
  return y + z;
}
```

Extracted code

```
let wmpn_cmp (x y: ptr uint64)
             (sz: int32): int32
= let ref i = sz in
  while i ≥ 1 do
    i ← i - 1;
    let lx = x[i] in
    let ly = y[i] in
    if lx ≠ ly then
      if lx > ly
      then return 1
      else return (-1)
  done;
  0
```

```
int32_t wmpn_cmp(uint64_t * x,
                 uint64_t * y,
                 int32_t sz)
{
  int32_t i;
  uint64_t lx, ly;
  i = sz;
  while (i >= 1) {
    i = i - 1;
    lx = x[i];
    ly = y[i];
    if (!(lx == ly)) {
      if (lx > ly) {
        return 1;
      } else {
        return -1;
      }
    }
  }
  return 0;
}
```

Long division: naïve algorithm

One iteration of the main loop

Goal: compute most significant limb of the quotient

a_0	a_1	$\ldots$	a_{m-3}	a_{m-2}	a_{m-1}

dividend/partial remainder: length m

d_0	$\ldots$	d_{n-1}

normalized divisor:
length n

?	?	$\ldots$	?	?

quotient: length $m-n$

Long division: naïve algorithm

One iteration of the main loop

Goal: compute most significant limb of the quotient

1. Estimate the most significant quotient limb (with a short division)

$$\underbrace{\begin{array}{|c|c|c|c|c|c|}\hline a_0 & a_1 & \dots & a_{m-3} & a_{m-2} & a_{m-1} \\ \hline\end{array}}_{\text{dividend/partial remainder: length } m}$$

$$\underbrace{\begin{array}{|c|c|c|}\hline d_0 & \dots & d_{n-1} \\ \hline\end{array}}_{\substack{\text{normalized divisor:} \\ \text{length } n}}$$

$$\underbrace{\begin{array}{|c|c|c|c|c|}\hline ? & ? & \dots & ? & ? \\ \hline\end{array}}_{\text{quotient: length } m-n}$$

$$\hat{q} = \overline{a_{m-2}a_{m-1}}/d_{n-1}$$

Long division: naïve algorithm

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Goal: compute most significant limb of the quotient

1. Estimate the most significant quotient limb (with a short division)
2. Multiply by the divisor, subtract the product from the dividend

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$$\underbrace{\begin{array}{|c|c|c|c|c|}\hline ? & ? & \dots & ? & \hat{q} \\ \hline\end{array}}_{\text{quotient: length } m-n}$$

$$\overline{a'_0 a'_1 \dots a'_{m-2}} = \overline{a} - \hat{q} \times \overline{d}$$

Long division: naïve algorithm

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$$\underbrace{\boxed{d_0}\boxed{\dots}\boxed{d_{n-1}}}_{\text{normalized divisor: length } n}$$

$$\underbrace{\boxed{?}\boxed{?}\boxed{\dots}\boxed{?}\boxed{\hat{q}}}_{\text{quotient: length } m-n}$$

$$\overline{a'_0 a'_1 \dots a'_{m-2}} = \overline{a} - \hat{q} \times \overline{d}$$

if $\hat{q}$ is right $a'_{m-1} = 0$

Long division: naïve algorithm

One iteration of the main loop

Goal: compute most significant limb of the quotient

1. Estimate the most significant quotient limb (with a short division)
2. Multiply by the divisor, subtract the product from the dividend
3. If the quotient is too large, adjust it

$$\underbrace{\boxed{a'_0}\boxed{a'_1}\boxed{\ldots}\boxed{a'_{m-3}}\boxed{a'_{m-2}}\boxed{a_{m-1}}}_{\text{dividend/partial remainder: length } m}$$

$$\underbrace{\boxed{d_0}\boxed{\ldots}\boxed{d_{n-1}}}_{\substack{\text{normalized divisor:}\\ \text{length } n}}$$

$$\underbrace{\boxed{?}\boxed{?}\boxed{\ldots}\boxed{?}\boxed{\hat{q}}}_{\text{quotient: length } m-n}$$

$$\hat{q} \leftarrow \hat{q} - 1 \qquad \overline{a'} \leftarrow \overline{a'} + \overline{d}$$

until it works...

Long division: naïve algorithm

One iteration of the main loop

Goal: compute most significant limb of the quotient

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Optimization: 3-by-2 division (Möller & Granlund 2011)

Goal: better estimate of the quotient, simplify adjustment step

$$\underbrace{\boxed{a_0}\boxed{a_1}\boxed{\ldots}\boxed{a_{m-3}}\boxed{a_{m-2}}\boxed{a_{m-1}}}_{\text{dividend/partial remainder: length } m}$$

$$\underbrace{\boxed{d_0}\boxed{\ldots}\boxed{d_{n-2}}\boxed{d_{n-1}}}_{\text{normalized divisor: length } n}$$

$$\hat{q} = \overline{a_{m-3}a_{m-2}a_{m-1}} / \overline{d_{n-2}d_{n-1}}$$

$$\overline{r_0 r_1} = \overline{a_{m-3}a_{m-2}a_{m-1}} - \hat{q} \cdot \overline{d_{n-2}d_{n-1}}$$

Adjustment: at most one step, and only if $r_1 = 0 \Rightarrow$ very unlikely

same divisor at each iteration $\Rightarrow$ 3-by-2 division uses a precomputed pseudo-inverse and no division primitive

Implementation trick: long subtraction

$$\underbrace{\boxed{a_0}\boxed{a_1}\boxed{\dots}\boxed{\color{red}a_{m-3}}\boxed{\color{red}a_{m-2}}\boxed{\color{red}a_{m-1}}}_{\text{dividend/partial remainder: length } m} \qquad \hat{q} = \overline{a_{m-3}a_{m-2}a_{m-1}} / \overline{d_{n-2}d_{n-1}}$$

$$\underbrace{\boxed{d_0}\boxed{\dots}\boxed{\color{red}d_{n-2}}\boxed{\color{red}d_{n-1}}}_{\substack{\text{normalized divisor:} \\ \text{length } n}} \qquad \overline{r_0 r_1} = \overline{a_{m-3}a_{m-2}a_{m-1}} - \hat{q} \cdot \overline{d_{n-2}d_{n-1}}$$

$$\overline{a'_0 a'_1 \dots a'_{m-2}} = \overline{a_0 \dots a_{m-3}a_{m-2}a_{m-1}} - \beta^{m-n-1}\hat{q} \times \overline{d_0 \dots d_{n-2}d_{n-1}}$$

but we already have $\overline{a_{m-3}a_{m-2}a_{m-1}} - \hat{q} \times \overline{d_{n-2}d_{n-1}} = \overline{r_0 r_1}$

$\Rightarrow$ subtraction over length $n-2$ instead of n, then propagate borrow

Final algorithm

```
while (i > 0) do
  i ← i - 1;
  xp ← C.incr xp (-1);
  let xd = C.incr xp mdn in
  let xp1 = xp[1] in
  if [@extraction:unlikely] (x1 = dh && xp1 = dl)
  then ...
  else begin
    let xp0 = xp[0] in
    (ql, x1, x0) ← div3by2_inv x1 xp1 xp0 dh dl v;
    let cy = wmpn_submul_1 xd y (sy - 2) ql in
    let cy1 = if (x0 < cy) then 1 else 0 in
    x0 ← sub_mod x0 cy;
    let cy2 = if (x1 < cy1) then 1 else 0 in
    x1 ← sub_mod x1 cy1;
    xp[0] ← x0;
    if [@extraction:unlikely] (cy2 ≠ 0)
    then begin     (* cy2 = 1  *)
      let c = wmpn_add_n_in_place xd y (sy - 1) in
      x1 ← add_mod x1 (add_mod dh c);
      ql ← ql - 1;
    end;
    qp ← C.incr qp (-1);
    qp[0] ← ql;
  end;
done;
```

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    let cy1 = if (x0 < cy) then 1 else 0 in
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```

3-by-2 division

Final algorithm

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  end;
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```

3-by-2 division

Shortened long subtraction

Final algorithm

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3-by-2 division

Shortened long subtraction

One-step adjustment

Final algorithm

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    end;
    qp ← C.incr qp (-1);
    qp[0] ← ql;
  end;
done;
```

3-by-2 division

Shortened long subtraction

One-step adjustment

Proof effort

div3by2_inv: 1kloc

long division: 2kloc

WhyMP

Objectives

- Verified C library
- Compatible with GMP
- Performances comparable to GMP, for numbers up to a certain size
- Contains a large subset of algorithms from the `mpn` and `mpz` layers

WhyMP

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- Compatible with GMP
- Performances comparable to GMP, for numbers up to a certain size
- Contains a large subset of algorithms from the `mpn` and `mpz` layers

Challenges

- Understand the algorithms
- Preserve GMP's implementation tricks

Noteworthy algorithms

Toom-Cook multiplication

- Divide-and-conquer multiplication algorithm in $O(n^k)$, $k \approx 1.58$
- Two mutually recursive variants:
  - Toom-2: split each operand in 2 parts ($\sim$ Karatsuba)
  - Toom-2.5: split large operand in 3 parts and small in 2
- Main challenge: aliasing

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Modular exponentiation

- Square-and-multiply exponentiation algorithm
- Montgomery reduction optimization: no division in the main loop
- Main challenge: formalization of mathematical concepts

Noteworthy algorithms

Square root of a 64-bit integer

- Hand-coded fixed-point arithmetic
- Newton iteration
- Converges in two steps for all inputs
- Main challenge: modeling fixed-point arithmetic

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`mpz` layer

Wrapper around the `mpn` layer, keeps track of number signs and sizes

- User-facing layer of GMP
- Not much arithmetic, but challenging aliasing combinatorics
- Main challenge: custom memory model required

Proof effort

Proof effort

- 22000 lines of WhyML code
  - 8000 of programs
  - 14000 of spec and assertions
- almost only automated provers
- total proof replay time: $\sim$ 1hr
- extracted C code: $\sim$ 5000 lines
- $\sim$ 100 functions, 50 of which are part of GMP's API

addition	1000
subtraction	1000
mul (naïve)	700
mul (Toom)	2400
division	4500
helper lemmas	300
reflection	1700
shifts	1000
square root	1600
modular exponentiation	1700
util	200
base conversions	2000
mpz	3600

Trusted code base

- Axioms, WhyML model of C
- Why3 verification condition computation
- Automated theorem provers
- Compilation from WhyML to C
- Handwritten arithmetic primitives

Arithmetic primitives

GMP uses handcoded assembly primitives:

- for basic operations:
  - $64 \times 64 \to 128$ bit multiplication
  - 128 by 64 bit division
- for critical large integer routines:
  - same-size addition $\square\square\square\square\square + \square\square\square\square\square$
  - *n*-by-1 multiplication $\square\square\square\square\square \times \square$

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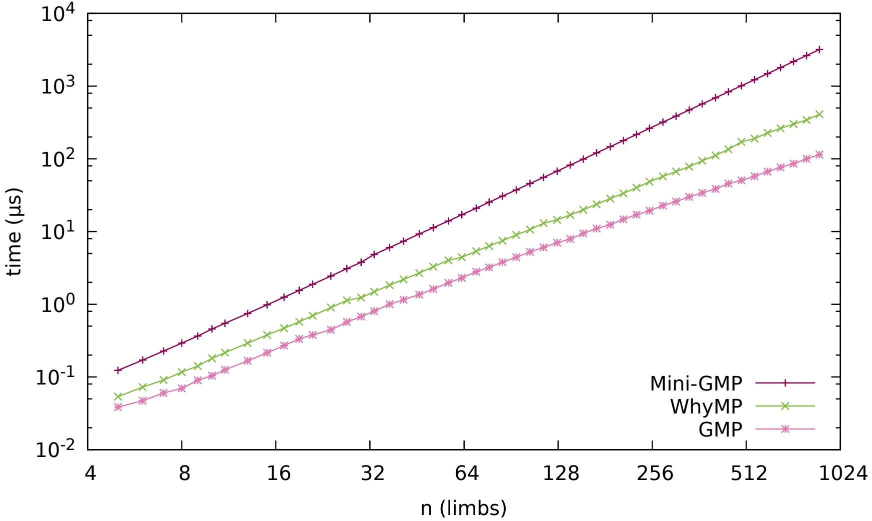
Options for WhyMP

- Trust the assembly primitives $\Rightarrow$ should we? which ones?
- Verified 64-bit C primitives $\Rightarrow$ much slower
- Compromise: handcoded C basic ops using 128-bit compiler support

Comparison with GMP ($n \times n$ multiplication)



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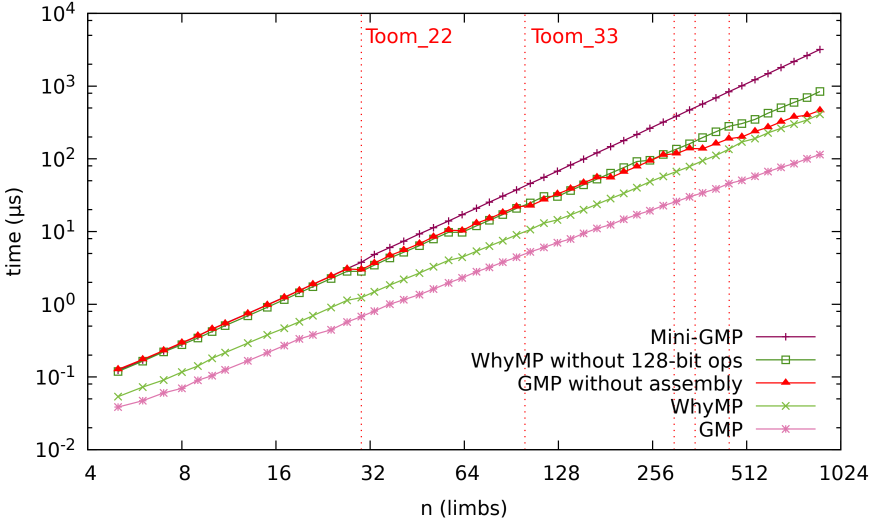

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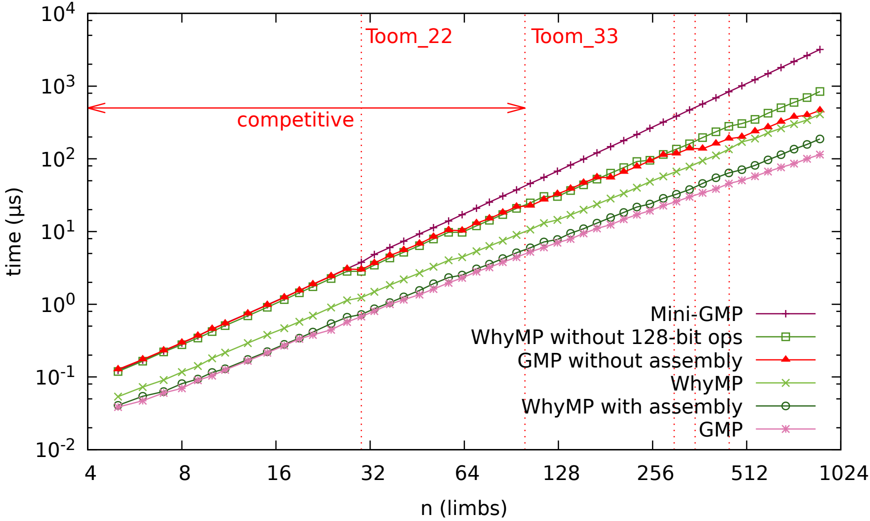

Verification of C programs with Why3

Contributions

- Memory model of the C language
- Straightforward extraction to C
- Works on more than GMP! (Contiki's ring buffer, cursors...)

$\Rightarrow$ Idiomatic, correct-by-construction C programs verified with Why3

Perspectives

- Memory model improvements:
  - support for C stack allocation
  - better alias handling
- Formalization of the correctness of the extraction mechanism

WhyMP

- Compatible with GMP (50 exported functions)
- Reasonable performance
- Formally verified! $\Rightarrow$ minor bug found in GMP
- Preserves most of GMP's implementation tricks

What remains to be done

- Exhaustivity: implement missing operations
- Cryptography functions, number theory functions
- Assembly code verification

`https://gitlab.inria.fr/why3/whymp`