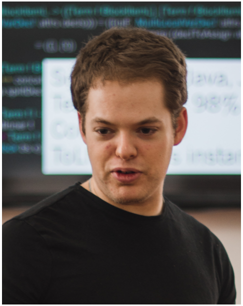
# Abstract

Dans cet exposé je vais vous montrer comment implémenter un effet de non-déterminisme angélique (par exemple l'opérateur amb) directement en OCaml, en utilisant seulement de l'état modifiable et des exceptions. Cette implémentation peut s'étendre pour obtenir les continuations délimitées !
Il n'est pas nécessaire de connaître déjà le non-déterminisme ou les continuations pour suivre l'exposé, qui commencera (pour la culture) par un petit tour d'horizon des notions d'effet en langages de programmation: style direct et indirect, monades, la réflexion monadique de Filinski, et les *effect handlers*.

# Tout réussir en répétant beaucoup

James Koppel, **Gabriel Scherer**, Armando Solar-Lezama

May 22, 2018



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We are going to:

- do something impossible about effects

Something impossible: pure OCaml (direct-style) implementation of *nondeterminism*, which extends to *delimited continuations*.

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We are going to:

- do something impossible about effects
- in a disappoingly simple way (Jimmy's neat trick)
- proved correct (by me, in the easy case)
- starting with useful background (for you)

Something impossible: pure OCaml (direct-style) implementation of *nondeterminism*, which extends to *delimited continuations*.

# Section 1

## Background on effects

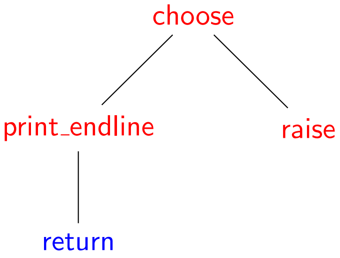
The core of programming:
(de)constructing values + performing function calls.

The rest is *side effects*:

- state
- Input/Output
- exceptions
- non-determinism
- system calls
- continuations
- ...

# a computation tree

**if** choose [**true**; **false**]
**then** (print_endline "it worked"; 42)
**else** raise (Failure "oops")




(computation goes down and up again)

# Direct and indirect style

```
let rec enum_nqueens i qs =
  if i = n then qs else
    let q = choose (List.filter (okay qs) range) in
    enum_nqueens (i+1) (q :: qs)
```

# Direct and indirect style

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let rec enum_nqueens i qs =
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let rec enum_nqueens i qs =
  if i = n then [qs]
  else List.fold_left
         (fun sols q → if not (okay qs q) then sols
                        else enum_nqueens (i+1) (q :: qs) @ sols)
         [] range
```

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         [] range

let rec enum_nqueens i qs =
  if i = n then ListMonad.return qs else
    ListMonad.bind (List.filter (okay qs) range) @@ fun q →
    enum_nqueens (i + 1) (q :: qs)
```

# Filinski's monadic reflection (1994)

```
module Reflect (M : Monad) : sig
  val reflect : 'a M.t → 'a
  val reify : (unit → 'a) → 'a M.t
end
```

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module Reflect (M : Monad) : sig
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module Choice = Reflect(ListMonad)

let rec enum_nqueens i qs =
  if i = n then qs else
    let q = Choice.reflect (List.filter (okay qs) range) in
    enum_nqueens (i+1) (q :: qs)

let solutions = Choice.reify (fun () → enum_nqueens 0 [])
```

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let solutions = Choice.reify (fun () → enum_nqueens 0 [])
```

Possible in any language with delimited continuations (shift/reset).

# Effect handlers (Plotkin and Pretnar, 2009)

```
effect Choose : 'a list → 'a

let rec enum_nqueens i qs =
  if i = n then qs
  else
    let q = perform (Choose (List.filter (okay qs) range)) in
    enum_nqueens (i + 1) (q :: qs)

let with_choice m =
  match m () with
  | r → [r]
  | effect (Choose li) k →
    List.flatten (List.map (fun v → continue k v) li)

let solutions = with_choice (fun () → enum_nqueens 0 [])
```

(Implemented in Multicore OCaml.)

```
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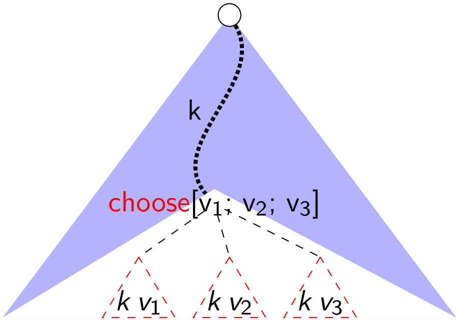
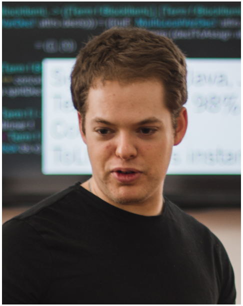


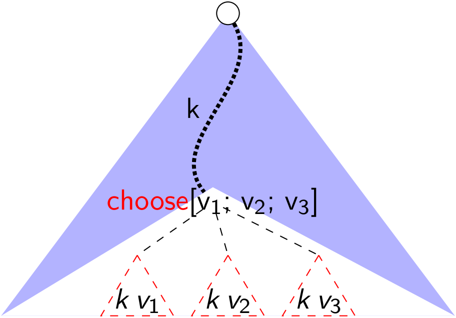

(uses continuations again)

# Section 2

## Jimmy's neat trick

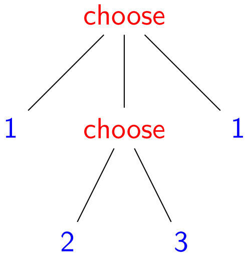


**val** choose : 'a list → 'a
**val** with_choice : (unit → 'a) → 'a list



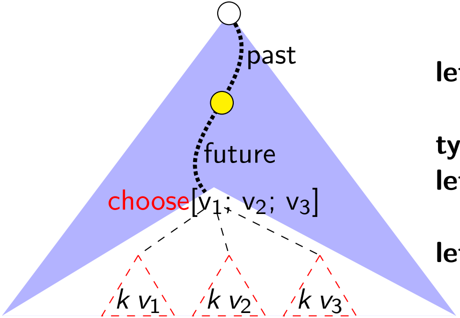

Jimmy's trick: if we can't *capture* k, just *replay* it.

```
with_choice begin fun () →
  if choose [true; false; true] then 1
  else
    if choose [true; false] then 2 else 3
end
```




On replay, remember the value

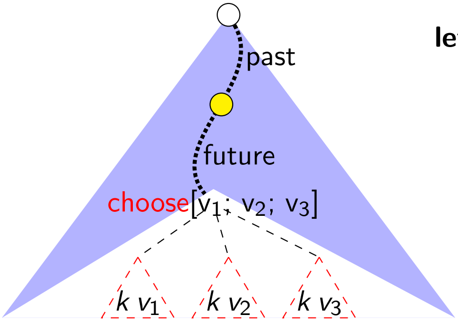
# Setup (1/3)




```
type idx = int * int
let start_idx xs = (0, List.length xs)
let next_idx (k, len) =
  if k + 1 = len then None
  else Some (k + 1, len)
let get xs (k, len) = List.nth xs k

type 'a stack = 'a list ref
let push stack x =
  stack := x :: !stack
let pop stack = match !stack with
  | [] -> None
  | x::xs -> stack := xs; Some x
```

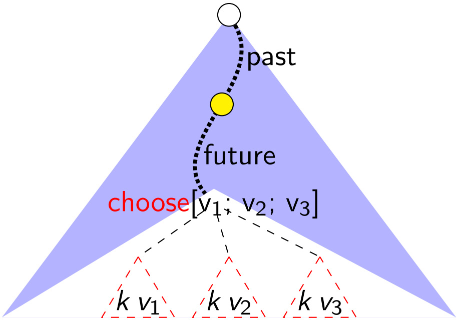
# choose (2/3)




```
let past = ref []
let future = ref []
exception Empty

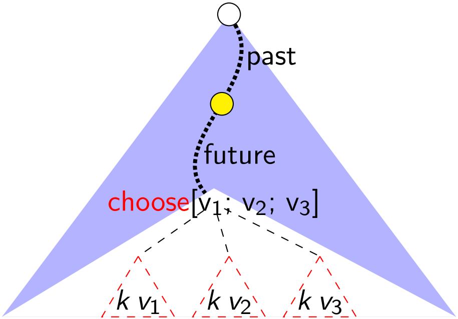
let choose = function
  | [] → raise Empty
  | xs →
    let i = match pop future with
    | None → start_idx xs
    | Some i → i
    in
    push past i;
    get xs i
```

# with_choice (3/3)



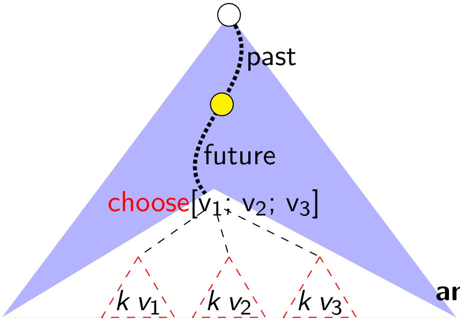

```
let rec with_choice f = loop f []
and loop f acc =
  let r =
    try [f ()] with Empty → [] in
  let acc = r @ acc in
```

# with_choice (3/3)




```
let rec with_choice f = loop f []
and loop f acc =
  let r =
    try [f ()] with Empty → [] in
  let acc = r @ acc in
  match next_path !past with
  | None → List.rev acc
  | Some path →
    past := [];
    future := List.rev path;
    loop f acc
```

# with_choice (3/3)

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    try [f ()] with Empty → [] in
  let acc = r @ acc in
  match next_path !past with
  | None → List.rev acc
  | Some path →
    past := [];
    future := List.rev path;
    loop f acc
and next_path = function
  | [] → None
  | i::is →
    match next_idx i with
    | Some i' → Some (i'::is)
    | None → next_path is
```

# Delimited continuations

Jimmy extended this idea to implement *delimited continuations*.

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*Not* in this talk!

https://arxiv.org/abs/1710.10385

# Section 3

## Non-determinism: correctness proof

# Continuation machines

$$\boxed{(t, K, s, R)} \qquad (t, \mathsf{halt}, \emptyset, \emptyset)$$

$$
\begin{array}{l}
t, u ::= \\
\quad \mid x, y, z \ldots \\
\quad \mid n \in \mathbb{N} \\
\quad \mid \mathsf{S}\ t \\
\quad \mid \mathsf{let}\ x\ =\ t\ \mathsf{in}\ t' \\
\quad \mid \mathsf{choose}\ x\ y
\end{array}
\qquad
\begin{array}{l}
K ::= \\
\quad \mid \mathsf{S}\ K \\
\quad \mid \mathsf{let}\ x\ =\ \square\ \mathsf{in}\ (t, K) \\
\quad \mid \mathsf{halt}
\end{array}
$$

$$
\begin{array}{lcl}
s & ::= & \emptyset \mid (t, K).s \\
R & ::= & \emptyset \mid n.R
\end{array}
$$

# Continuation machines

$$\boxed{(t, K, s, R)} \qquad (t, \mathsf{halt}, \emptyset, \emptyset)$$

$$
\begin{array}{lcl}
(\mathsf{S}\ t, K, s, R) & \to & (t, \mathsf{S}\ K, s, R) \\
(n, \mathsf{S}\ K, s, R) & \to & (n+1, K, s, R)
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(\mathsf{let}\ x\ =\ t\ \mathsf{in}\ t', K, s, R) & \rightarrow & (t, (\mathsf{let}\ x\ =\ \square\ \mathsf{in}\ (t', K)), s, R)
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(n, \mathsf{S}\ K, s, R) & \to & (n+1, K, s, R) \\
(\mathsf{let}\ x = t\ \mathsf{in}\ t', K, s, R) & \to & (t, (\mathsf{let}\ x = \square\ \mathsf{in}\ (t', K)), s, R) \\
(n, \mathsf{let}\ x = \square\ \mathsf{in}\ (t', K), s, R) & \to & (t'[x \leftarrow n], K, s, R)
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(n, \mathsf{let}\ x = \square\ \mathsf{in}\ (t', K), s, R) & \rightarrow & (t'[x \leftarrow n], K, s, R)
\end{array}$$

$$(\mathsf{choose}\ n_1\ n_2, K, s, R) \quad \rightarrow \quad (n_1, K, (n_2, K).s, R)$$

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\end{array}
$$

$$
\begin{array}{lcl}
(\mathsf{choose}\ n_1\ n_2, K, s, R) & \rightarrow & (n_1, K, (n_2, K).s, R) \\
(n, \mathsf{halt}, (n', K).s, R) & \rightarrow & (n', K, s, n.R)
\end{array}
$$

# History machines

$$\boxed{(t, K, P, F, R)_u} \qquad (t, \mathsf{halt}, \emptyset, \emptyset, \emptyset)_t$$

$$i \quad ::= \quad 1 \mid 2 \qquad \begin{array}{lcl} P & ::= & \emptyset \mid P.i \\ F & ::= & \emptyset \mid i.F \end{array}$$

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(\mathsf{let}\ x\ =\ t\ \mathsf{in}\ t', K, P, F, R)_u & \rightarrow (t, \mathsf{let}\ x\ =\ \square\ \mathsf{in}\ (t', K), P, F, R)_u \\
(n, \mathsf{let}\ x\ =\ \square\ \mathsf{in}\ (t', K), P, F, R)_u & \rightarrow (t'[x \leftarrow n], K, P, F, R)_u
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(n, \mathsf{let}\ x\ =\ \square\ \mathsf{in}\ (t', K), P, F, R)_u & \rightarrow (t'[x \leftarrow n], K, P, F, R)_u
\end{array}$$

$$(\mathsf{choose}\ n_1\ n_2, K, P, \emptyset, R)_u \quad \rightarrow \quad (\mathsf{choose}\ n_1\ n_2, K, P, 1.\emptyset, R)_u$$

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(\mathsf{choose}\ n_1\ n_2, K, P, \emptyset, R)_u & \rightarrow & (\mathsf{choose}\ n_1\ n_2, K, P, 1.\emptyset, R)_u \\
(\mathsf{choose}\ n_1\ n_2, K, P, (i.F), R)_u & \rightarrow & (n_i, K, (P.i), F, R)_u
\end{array}
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\end{array}$$

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(n, \mathsf{halt}, P, \emptyset, R)_u & \rightarrow & (u, \mathsf{halt}, \emptyset, P{+}1, n.R)_u
\end{array}$$

$$\begin{array}{lcl}
P.1{+}1 & \stackrel{\text{def}}{=} & P.2 \\
P.2{+}1 & \stackrel{\text{def}}{=} & P{+}1
\end{array}$$

# Proof: combined machines

$$\boxed{(t, K_P, F, s, R)_u} \qquad (t, \mathsf{halt}_\emptyset, \emptyset, \emptyset, \emptyset)_t$$

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$$\begin{array}{lcl}
(\text{choose } n_1\ n_2, K_P, \emptyset, s, R)_u & \rightarrow & (n_1, K_{P.1}, \emptyset, (n_2, K_{P.2}).s, R)_u \\
(\text{choose } n_1\ n_2, K_P, i.F, s, R)_u & \rightarrow & (n_i, K_{P.i}, F, s, R)_u \\
(n, \mathsf{halt}_P, \emptyset, (n', K_{P'}).s, R)_u & \rightarrow & (n', K_{P'}, \emptyset, s, n.R)_u
\end{array}$$

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\end{array}$$

# Proof: timeline and replay

$$(n, \mathsf{halt}, P, \emptyset, R)_u \rightarrow (u, \mathsf{halt}, \emptyset, P{+}1, n.R)_u$$
$$(n, \mathsf{halt}_P, \emptyset, (n', K_{P'}).s, R)_u \rightarrow (n', K_{P'}, \emptyset, s, n.R)_u$$

$$(n, \mathsf{halt}_P, \emptyset, (n', K_{P'}).s, R)_u \rightarrow (u, \mathsf{halt}_\emptyset, P', s, n.R)_u \rightarrow^* (n', K_{P'}, \emptyset, s, n.R)_u$$

# Proof: timeline and replay

$$(n, \text{halt}, P, \emptyset, R)_u \rightarrow (u, \text{halt}, \emptyset, P{+}1, n.R)_u$$
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Timeline Invariant:

$$P' = P{+}1$$

$$(\text{choose } n_1 \; n_2, K_P, \emptyset, s, R)_u \rightarrow (n_1, K_{P.1}, \emptyset, (n_2, K_{P.2}).s, R)_u$$

# Proof: timeline and replay

$$
\begin{array}{lcl}
(n, \text{halt}, P, \emptyset, R)_u & \to & (u, \text{halt}, \emptyset, P{+}1, n.R)_u \\
(n, \text{halt}_P, \emptyset, (n', K_{P'}).s, R)_u & \to & (n', K_{P'}, \emptyset, s, n.R)_u
\end{array}
$$

$$(n, \text{halt}_P, \emptyset, (n', K_{P'}).s, R)_u \to (u, \text{halt}_\emptyset, P', s, n.R)_u \to^* (n', K_{P'}, \emptyset, s, n.R)_u$$

Timeline Invariant:

$$P' = P{+}1$$

$$(\text{choose } n_1 \ n_2, K_P, \emptyset, s, R)_u \quad \to \quad (n_1, K_{P.1}, \emptyset, (n_2, K_{P.2}).s, R)_u$$

Replay Theorem:

$$\text{replay}(n, K_P, F, s, R)_u \stackrel{\text{def}}{=} (u, \text{halt}_\emptyset, (P.F), s, R)_u$$

$$(t, \text{halt}_\emptyset, \emptyset, \emptyset)_t \to^* c \quad \Longrightarrow \quad \text{replay}(c) \to^*_{\text{pure}} c$$

(Witty transition slide)

# Section 4

## Benchmarks!

# Worst case is very bad

```
with_choice (fun () →
  let v = long_pure_computation () in
  let i = choose [0; 1; 2; 3; 4; 5; 6; 7; 8; 9] in
  (i, v)
)
```

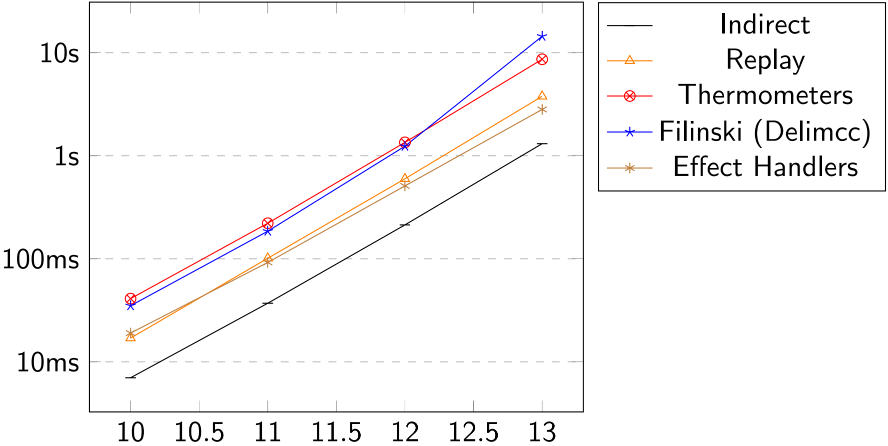
# N queens

```
let n = int_of_string Sys.argv.(1)
let range = Array.init n (fun i → i) |> Array.to_list

let okay qs q =
  let rec okay i c = function
    | [] → true
    | x::xs →
      c <> x && (c−x) <> i && (c−x) <> −i && okay (i+1) c xs
  in okay 1 q qs

let rec enum_nqueens i qs =
  if i = n then qs else
    let q = choose (List.filter (okay qs) range) in
    enum_nqueens (i+1) (q :: qs)

let nb_sols = List.length (with_choice (fun () → enum_queens 0 []))
```




| | **10** | **11** | **12** | **13** |
|---:|---:|---:|---:|---:|
| **Indirect** | 0.007s | 0.037s | 0.213s | 1.308s |
| **Replay** | 0.017s | 0.101s | 0.597s | 3.768s |
| **Therm.** | 0.041s | 0.221s | 1.347s | 8.621s |
| **Filinski (Delimcc)** | 0.035s | 0.185s | 1.236s | 14.412s |
| **Effect Handlers (Multicore OCaml)** | 0.019s | 0.092s | 0.509s | 2.81s |
| **Prolog search (SWI-Prolog)** | 0.611s | 2.997s | 17.616s | |

Thanks. Any questions?

```
queens(N, N, L, L).
queens(N, I, L, Res) :-
   I < N,
   choose_okay_in_range(0, N, C, L),
   I1 is I+1,
   queens(N, I1, [C|L], Res).

choose_okay_in_range(I, N, I, L) :- I < N, okay(1, I, L).
choose_okay_in_range(I, N, C, L) :-
   I < N, I1 is I+1, choose_okay_in_range(I1, N, C, L).

okay(_, _, []).
okay(I, C, [X|XS]) :-
   C =\= X, (C-X) =\= I, (X-C) =\= I, I1 is I+1, okay(I1, C, XS).

count(N, Count) :- aggregate_all(count, queens(N, 0, [], L), Count).
```